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Real Exchange Rate Dynamics and the Financial Theory of the Trading Firm

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I. Introduction

There is considerable controversy among international economists as to the nature of real exchange rate dynamics.¹ This controversy centers on two issues. First, economists disagree on whether the real exchange rate changes are transitory or permanent.² Second, they disagree on whether these changes belong to a distribution whose continuous-time limit is a diffusion process or the sum of a diffusion process and a jump process.³ In this paper, we investigate how real exchange rate dynamics affect the investment, financing and hedging policies of the trading firm.⁴ We show that the relevance of real exchange rate dynamics depends crucially on whether projects are long- or short-lived, on whether they have payoffs which are linear in the current spot real exchange rate, and on which type of financial instruments are available.

Empirical researchers who regress the logarithm of the current real exchange rate on the logarithm of the previous month real exchange rate and a constant cannot reject the hypothesis that the regression coefficient for the logarithm of the previous month real exchange rate is close to one or one. In the long run, however, a regression coefficient of, for instance, .95 has substantially different implications for real exchange rate dynamics than a coefficient of one. A coefficient of one implies that one expects the long run value of the real exchange rate to be the same as today’s, while a coefficient of .95 leads to an expected value of one for the real exchange rate in the long run when the constant can be neglected.³ Because this empirical evidence implies that the expected rate of change of the real exchange rate is close to zero in the short run, it suggests that the value of long-lived projects

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is more sensitive to one's assumptions about real exchange rate dynamics than the value of short-lived projects.

Our analysis shows, however, that the valuation of short-lived projects can be extremely sensitive to one's assumptions about real exchange rate dynamics when the current value of these projects is a nonlinear function of the current real exchange rate. An example of such a project is one where the decision to export a product is taken at a future date and depends on the real exchange rate at that future date. This is because, for such projects, the expectation of the projects' payoffs depends crucially on the second moment of the distribution of the real exchange rate. Even if one takes the local variance of real exchange rate changes to be given, different assumptions about real exchange rate dynamics can have sharply different implications for the second moment of the distribution of future real exchange rates. Somewhat surprisingly, however, the value of the firm is in some cases an increasing function of the conditional variance of the real exchange rate, so that increased real exchange rate volatility can make the firm better off.6

In this paper, the trading firm has an optimal amount of debt because of the deductibility of interest costs from taxable income and of the existence of bankruptcy costs. After-tax income is assumed to be a concave function of pre-tax income so that, following Smith/Stulz (22), the firm has an optimal hedging policy. The optimal hedging and financing policies are derived when both individuals and corporations pay taxes. We show that the firm's optimal financing policy depends crucially on whether the real exchange rate dynamics include a jump process or not, except when the value of the trading firm in the absence of debt and hedging is a linear function of the current real exchange rate. The firm's optimal financing and hedging policies are not constrained by the investment opportunity set if there exists a self-financing portfolio whose value at maturity equals the firm's income from the sale of its output. The number of traded securities required for the firm to construct such a self-financing portfolio depends crucially on the real exchange rate dynamics. For instance, if the real exchange rate follows a mean-reverting process and does not have a continuous sample path, the firm may not be able to follow its optimal financing and hedging policies unless European options on the real exchange rate with an appropriate maturity are traded on financial markets. However, if the real exchange rate has a continuous sample path and follows a random walk, the firm can implement its optimal financing and hedging policies if there exists, at all points in time, at least one asset whose value is an appropriately differentiable function of the real exchange rate and time only.

The plan of the paper is as follows. In Section II, we present the rudiments of a two-country model in which the real exchange rate changes stochastically over time. In Section III, we consider the simplest capital budgeting problem that a trading firm is confronted with, i.e., the discounting of the revenue from the sale abroad of a fixed number of units of the product. In Section IV, we study capital budgeting problems which involve options whose value depends on the real exchange rate. Section V solves for the optimal debt and hedging policies of the trading firm.
II. The Economy

In a world economy in which the prices of all goods, irrespective of where they are sold, are the same when expressed in a common numeraire, the real exchange rate can change over time only because consumption tastes differ across countries. In this case, changes in the real exchange rate are brought about by changes in relative prices and the effects of changes in the real exchange rate on a firm’s investment and financing policies have little to do with the fact that there is more than one country. Consequently, to make the real exchange rate directly relevant for the firm’s policies, we have to assume that the real exchange rate changes over time because goods sell for different prices in different countries.

In the international economics literature, it is common to assume that some goods are not traded internationally. In general, changes in the relative price of these goods have far-reaching effects both in the economy in which they are sold and in economies in which they are not available. Some goods can be non-traded goods simply because it takes time to transport them from one country to another. In this paper, we follow this tradition and assume that there is only one consumption good and that international trade in that good takes time. Since it matters where a unit of the good is located, the fact that the same good is sold at different prices in different countries at a given point in time does not imply that there exist arbitrage opportunities.

Production of the consumption good requires time and a capital good. The capital good is assumed to be internationally traded, so that its price in domestic currency is the same everywhere. This assumption reduces the number of prices we have to keep track of and reflects the widespread belief that the law of one price holds up better for intermediate than for consumption goods. We denote respectively by $P$ and $p$ the domestic currency price of the capital good and the price of the consumption good relative to the price of the capital good in the domestic country. We use asterisks to denote the same prices abroad. To focus the discussion squarely on the implications for the trading firm of alternative real exchange rate dynamics, we assume that $P = p = 1$. Therefore, the real exchange rate, $x$, is equal to the price abroad of the consumption good relative to the traded good, $p^*$. The foreign currency price of the foreign consumption good is $ep^*P^* = p^*$, since $eP^* = P = 1$. Importantly, our model can accommodate the well-known empirical result that the nominal and the real exchange rates are highly correlated. This will be the case whenever $P^*$ and $p^*$ are highly correlated.

We consider an economy in which asset and goods markets are open all the time and assume that $x$ follows:

$$\frac{dx}{x} = -(\lambda k + \mu + \delta \ln x)dt + \sigma dz + dq$$  \hspace{1cm} (1)

where $d_z$ is the increment of a standard Wiener process and $d(\tau)$ is an independent Poisson process with parameter $\lambda$. It is assumed that $\lambda$, $\mu$, $\delta$ and $\sigma$ are constant. $k = E(H - 1)$, where $H - 1$ is the random percentage rate of change in $x$ if the
Poisson event occurs and $E$ is the expectation operator over the random variable $H$. This process implies that the real exchange rate can never be negative and it has several other attractive features. Note first that when $\lambda = 0$ the real exchange rate follows a diffusion process. Further, if $\delta = 0$, the distribution of the rate of change of the real exchange rate does not depend on the current real exchange rate and the real exchange rate is a random walk. The real exchange rate follows a martingale when $\mu = -(1/2)\sigma^2$ and $\lambda = \delta = 0$. When $\delta > 0$ and $\mu = -\sigma^2/\delta$, the real exchange rate is expected to move towards one. It follows from this that our specification of the real exchange rate dynamics encompasses the benchmark cases discussed in the introduction.

In the analysis that follows, risk aversion induces second-order effects, so that little is lost if we assume that individuals are risk neutral. We consider a world economy in which asset markets are perfect, in the sense that there are no transaction costs, no restrictions to short-sales and that individuals and firms are price-takers. Until Section IV, we also assume that there are no taxes. Borrowing and lending at the risk-free rate is possible. The domestic risk-free rate, $r$, is assumed to be constant. With our assumptions, $r$ is both the nominal and the real rate of interest.

III. Valuation of Foreign Currency Receipts

In this section, we consider a domestic firm which has to decide at date $t$ whether to invest $\ell$ in domestic currency to produce one unit of the consumption good for sale at home or one unit of the same good for sale abroad at date $T$. It is assumed that the firm's decision affects neither the domestic nor the foreign relative price of the consumption goods. The analysis that follows is a partial equilibrium analysis, since no assumptions are made about the behavior of other firms. However, for the resulting aggregate equilibrium to be sensible, it must be the case that some firms differ in production costs or technology from the one we are considering here—for instance, some firms might not be able to export. Otherwise, it is possible for all production to be done either for the domestic or for the foreign market. The firm receives at date $T$ one unit of domestic currency if it sells the product at home and $P^*(T)p^*(T)$ units of foreign currency if it sells abroad. The period $T - t$ represents a production and transportation lag. Throughout the paper, it is assumed that the firm's managers choose projects and financing policies which maximize the current value of the firm.

The problem considered here is the most simple open-economy capital budgeting problem. However, many more complicated problems can be viewed as portfolios whose constituting elements are similar to the simple project considered here. It will be apparent that our main conclusions generalize to such portfolios. The traded good can be used as the numeraire and $r$ is the rate of interest in that
numeraire. With our assumptions, the domestic project has the highest net present value of the two projects if:

\[ e^{-r(0-k)} - 1 > e^{-r(T-t)}E(x(T)) - 1 \]  

(2)

Otherwise, the foreign project has the highest net present value. By assumption, the firm chooses the project with the highest positive net present value. For the real exchange rate to matter in the firm’s decision, the least valued project must be profitable. We assume that this is the case throughout the paper, so that the decision rule becomes that the foreign project is taken if the expected real exchange rate exceeds one:

\[ E(x(T)) > 1 \]  

(3)

Computing the expected value of the real exchange rate under the assumption that it follows a diffusion process with \( \mu = -\sigma^2/4\delta \), we get:

\[ E(x(T)) = x(t) \exp \left[ -\left( \log x(t) - \frac{\sigma^2}{4\delta} \right) \left( 1 - e^{-\delta s} \right) + \frac{\sigma^2}{4\delta} \left( 1 - e^{-2\delta s} \right) \right] \]  

(4)

where \( s = T - t > 0 \). With \( \delta = 0 \) and \( \mu = -(1/2)\sigma^2 \), \( E(x(T)) = x(t) \). With \( \delta > 0 \) and \( \mu = -\sigma^2/4\delta \), \( E(x(T)) > 1 \) if \( x(t) \) is greater than one. However, with the same values of \( \delta \) and \( \mu \), it is possible for \( E(x(T)) > 1 \) if \( x(t) \) is smaller than one, for some values of \( T < \infty \). To see this, note that collecting the terms which depend on \( \sigma^2 \) in equation (4) yields \( (\sigma^2/4\delta)(e^{-\delta s} - e^{-2\delta s}) \). This term is positive and does not depend on the \log of \( x(t) \). Hence, if \( x(t) \) is smaller than one, it is possible for \( E(x(T)) \) to exceed one when \( s \) is neither too small nor too large. However, one would expect \( E(x(T)) \) to be smaller than one when \( x(t) \) is smaller than one, since the speed of adjustment \( \delta \) is small for real exchange rates. This means that, in general, precise knowledge of \( \delta \) is irrelevant for this capital budgeting problem. This conclusion would not be changed if the firm could undertake many different projects of the type considered here that differ in their maturity date.

It is important, however, to recognize that if, in the long run, the real exchange rate converges to some value other than one, the capital budgeting decision becomes more complicated. To wit, suppose that, as \( T \to \infty \), \( x(T) \to r > 1 \). In this case, the firm always takes the foreign project if \( x(t) > 1 \). However, it can also make sense for the firm to produce the exported good when \( x(t) < 1 \), as it is possible for \( E(x(T)) \) to be larger than one when \( x(t) \) is below one. The firm is more likely to choose the foreign project when \( x(t) \) is smaller than one if (a) the project is long-lived, (b) \( \delta \) is large, and (c) \( x(t) \) is close to one.
Interestingly, the case in which it costs more to produce for one country than for the other is isomorphic to the case in which \( x(t) \) converges to some value other than one. To see this, suppose \( I^* \) is the cost of producing the product for the foreign market while \( I \) is the cost of producing it for the domestic market. In this case, the foreign project is taken if:

\[
E ( x ( T ) ) - 1 > e^r ( T - t ) ( I^* - I )
\]  

Hence, for the foreign project to dominate, the real exchange rate has to exceed one at time \( T \) by \( e^r ( T - t ) ( I^* - I ) \). If the cost of producing for abroad exceeds the cost of producing for the domestic market, the firm can find it unprofitable to produce for the foreign market even if \( x(t) \) exceeds one, as long as \( x(t) \) is not too large. In this case, a large \( \delta \) makes it less likely that the firm will choose to produce for the foreign market when \( x(t) \) exceeds one, since it means that the real exchange rate converges to one more quickly and hence is unlikely to be large enough at date \( T \).

If the real exchange rate follows a mixed diffusion-jump process, equation (9) gives the expected value of the real exchange rate if no jump takes place. If the distribution of the jump size does not affect the expected real exchange rate, equation (9) still yields the expected real exchange rate. This means that, in this case, the possibility of jumps in the path of the real exchange rate has no effect on the capital budgeting problem studied in this section.

IV. Valuation of Contingent Foreign Currency Receipts

In this section, we extend the capital budgeting problem discussed in the previous section to enable us to consider the case in which the value of the firm is a function of actions taken by the firm at a later date which depend on the real exchange rate at that date. We consider a problem that, despite its simplicity, has fairly general implications, since most projects can be viewed as portfolios of projects of the type considered in the previous section and of the type studied here. We assume that, at date \( t \), the firm decides whether to produce one unit of the consumption good. At date \( s^* \), the unit must be shipped either to the foreign country or to markets in the domestic country. Finally, at date \( T \), the unit is sold in the country to which it was shipped.

At date \( T \), the firm receives either one or \( x \) units of domestic currency depending on where the product is shipped at date \( s^* \) since, because \( e^{P^*} = p = p^* \), \( P^*p^* \) units of foreign currency are equal to \( x \) units of domestic currency. At date \( s^* \), the firm exports the product if \( E_{s^*}(x(T)) \) exceeds one. Hence, the present value of the project, \( V \), is:

\[
V = e^{-r(T-s^*)} E_{s^*}[\max ( E_{s^*}(x(T)) - 1, 0 )]
\]
To understand equation (6), note that the value of the project is equal to the value of producing for the domestic market plus the present value of the option to export the commodities abroad at a later date. The firm will choose to export the product if it expects at time \( t^* \) that the real exchange rate will exceed one when the commodities are sold at time \( T \). Hence, to value the project, one has to compute the value of a call option on the real exchange rate.

We consider first the case in which the real exchange rate follows a martingale. In this case, \( E_{\mathcal{F}}(x(T)) = x(t^*) \). As \( x(t^*) \) follows a lognormal distribution, we have that:

\[
e^{-r^*} E_{\mathcal{F}} \left[ \max(x(t^*) - 1, 0) \right] = e^{-r^*} x(t^*) \mathcal{N} \left( \frac{\ln x(t^*) + (\sigma^2/2)s}{\sigma \sqrt{s}} \right) - e^{-r^*} \mathcal{N} \left( \frac{\ln x(t^*) - (\sigma^2/2)s}{\sigma \sqrt{s}} \right) \tag{7}
\]

where \( s = t^* - t \). Inspection of equation \( (7) \) reveals that it yields the formula for a call option on the real exchange rate. This formula is similar to the formula for a call option on the nominal exchange rate.\(^{11} \) The option pricing analogy indicates that the firm’s project becomes more valuable as the variance of the rate of change of the real exchange rate increases. Hence, it is possible that the firm would not invest in the project when the real exchange rate is a constant while it does invest in the project when the real exchange rate is highly volatile. This is because a volatile real exchange rate enables the firm to sometimes sell the product at a higher price, but never forces the firm to sell the product abroad when it expects the foreign price to be low.

The case in which \( x(t) \) follows a mean-reverting process is now investigated. To compute the value of the project when the real exchange rate follows a mean-reverting process, note that the firm exports at date \( t^* \) if and only if \( x(t^*) \) exceeds one. Let \( y(t) \) satisfy:

\[
y(t) = x(t) \exp \left( \left[ -\log x(t) - \frac{\sigma^2}{4\delta} \right] \left( 1 - e^{-\delta v} \right) + \frac{\sigma^2}{4\delta} \left( 1 - e^{-\delta v} \right) \right) \tag{8}
\]

where \( v = T - t^* \). By construction, \( y(t^*) \) is the expected value of \( x(T) \) at time \( t^* \). Differentiating \( (1) \) using Ito’s lemma, we have:

\[
\frac{dy}{y} = \frac{dx}{x} e^{-\delta v} + \frac{1}{2} \left[ e^{-2\delta v} - e^{-\delta v} \right] \sigma^2 dt \tag{9}
\]
Note that \( \nu \) is constant. This implies that the growth rate of \( y \) is perfectly correlated with the growth rate of \( x \). Solving for the value of the option that is part of the project, we have:

\[
e^{-r(T-t)}E_{s} \left[ \text{Max}(x(T)-1,0) \right] = e^{(\nu \sigma -rs -\rho s)} y(t)N \left( \frac{\ln y(t) + \rho s + \delta/2}{\delta} \right) - e^{-r(T-t)} s_{*} y(t) \right)N \left( \frac{\ln y(t) + \rho s + \delta/2}{\delta} \right)
\]

where \( \rho \) is the expected growth rate of \( y(t) \), i.e., \( E(\ln(y(t))/y(t)) - e^{\rho s} \), and \( \sigma \) is the conditional variance of \( \ln(y(t))/y(t) \). Using Merton (1971), \( \sigma \) is given by:

\[
\sigma = e^{-2\delta} \frac{\sigma_{m}^{2}}{2\delta} \left( 1 - e^{-2\delta s} \right)
\]

To interpret equation 10, it is useful to note that when \( x(t) = 1 \), \( \rho \) is almost zero, at least for large values of \( s \), so that the formula for the value of the option differs from the case in which \( x(t) \) is a martingale and is equal to one mainly because \( \sigma \) differs from \( \sigma_{m}^{2} \). However, since \( \sigma \) is smaller than \( \sigma_{m}^{2} \), this means that the value of the option evaluated at \( x(t) = 1 \) is smaller when \( x(t) \) follows a mean-reverting process than when it follows a martingale. To discuss the drift effect on the value of the option, it is useful to consider the hypothetical case in which \( \sigma = \sigma_{m}^{2} \) and \( \nu = T \). In this case, \( y(t) = x(t) \). Note, however, that this case requires the local variance of the growth rate of the real exchange rate to differ when \( x(t) \) is a martingale and when it follows a mean-reverting process. If \( \sigma = \sigma_{m}^{2} \), the value of the two options differs to the extent that \( x(t) \) differs from one. If \( x(t) \) exceeds one, the option is more valuable if \( x(t) \) is a martingale because \( \rho \) is negative. The opposite is true if \( x(t) \) is smaller than one. As investors observe the real exchange rate continuously, they have no uncertainty about the local variance of the growth rate of the real exchange rate. Hence, for a given local variance of the growth rate of \( x(t) \), our discussion implies that the value of the option is greater when \( x(t) \) follows a martingale except for low values of \( x(t) \). The expected growth rate of \( x(t) \) is high if \( x(t) \) follows a mean-reverting process and its value is low. This effect can dominate the effect of a higher variance of \( \log(x(T)/x(t)) \) on the value of the option when \( x(t) \) is a martingale. When \( r \) differs from \( T \), the exercise price of the real exchange rate option may be lower if the real exchange rate follows a mean-reverting process than if it is a martingale. Consequently, the value of the real exchange rate option may be higher if \( c(t) \) follows mean-reverting process than if it is a martingale even for some values of \( c(t) \) greater than one.

Finally, we consider the case in which the real exchange rate follows a mixed
diffusion-jump process and has no drift. Suppose, for simplicity, that the jump size is lognormally distributed. Let $\lambda = \log(1 + k)$ and $J_n$ be equal to the product of $n$ jumps. In this case, $E_n(J_n) = \exp(n\gamma)$, where $E_n(J_n)$ is the expectation of $J_n$ conditional on knowing that $n$ jumps took place. Let $\Omega$ be the variance of the logarithm of $H$. The variance of the rate of change of the real exchange rate conditional on $n$ jumps is written $\nu_n^2 = (\sigma^2 + n\Omega^2/\delta)$ and we define $r_n$ to be equal to $-\lambda k + n\gamma/\delta$. With this notation, the value of the call option on the real exchange rate is:

$$C(x, 1, s) = \sum_{n=1}^{\infty} \frac{e^{-\nu_n^2}}{n!}(\lambda s)^n C_n(x, 1, s)$$

(12)

where $C_n(x, 1, s)$ is the value of a call option on the real exchange rate given that $n$ jumps took place and $\lambda' = \lambda(1 + k)$. $C_n(x, 1, s)$ is given by the Black/Scholes [1] formula with $r$ set equal to $r_n$ and $\sigma^2$ replaced by $\nu_n^2$.

Merton [16] compares the value of a call option when the stock price follows a lognormal diffusion process plus a jump process with lognormally distributed jumps to the value of a call option when the stock price follows a lognormal diffusion process. He argues that if the stock price follows a jump-diffusion process with lognormally distributed jump sizes and one mistakenly assumes that the stock price follows a lognormal diffusion process, one is likely to underestimate the value of the option when it is deep-in-the-money or deep-out-of-the-money, and when it has a short maturity. This means here that one is more likely to take a short-lived project if the real exchange rate follows a jump process so that, in this case, one’s assumptions about real exchange rate dynamics can affect the net present value of the project in non-trivial ways even when the project is short-lived.

V. Financing and Hedging Policies of the Trading Firm

In this section, we explore how different types of real exchange rate dynamics affect the trading firm’s financing and hedging policies. We do so in the context of a model which implies the existence of an interior solution for the firm’s debt-equity ratio. While the model is admittedly simple, it suffices to make the point that the firm’s value depends crucially on the degree to which markets are complete and the number of securities required to make markets complete is tied to the nature of the real exchange rate dynamics.

Although many variables are likely to affect the firm’s financing policies, in the following analysis we exclude all these variables but two, taxes and bankruptcy costs. Let $T$ be the taxable income of the trading firm at time $T$, i.e., when production is sold. $T$ is defined as the income from the product sold minus the cost of producing it and minus interest paid to bondholders. We consider an extremely stylized form of the tax code. If the trading firm cannot repay the principal and pay...
the interest, it is assumed that the trading firm first makes the interest payments and then starts to repay the principal. All debt is issued at par. Income net of taxes is $Y - \tau(Y)^{Y'}$, where $\tau(Y)$ is the tax rate. The tax rate is assumed to be an increasing function of $Y$ such that $\tau(Y^{Y_{\leq 0}}) = 0$, $\tau(Y^{Y_{\geq 0}}) > 0$, $\tau'(Y^{Y_{> 0}}) > 0$ and $\tau''(Y^{Y_{= 0}}) = 0$. A prime denotes a partial derivative, so that $\tau$ is the partial derivative of the tax rate with respect to taxable income. These assumptions imply that income net of taxes is a concave function of pre-tax income. To complete the model, we assume that individuals pay taxes on their income and that the marginal tax rate of the marginal investor is $\tau_{m}^{< r}$. Further, capital gains are not taxed and their losses are not deductible. Finally, we assume that if the trading firm is bankrupt, bankruptcy costs are equal to the minimum of the value of the firm gross of these costs and $B$.

We now consider the case discussed in Section III: the firm knows at date $t$ that it will receive $x(T)$ at date $T$. Consequently, the taxable income of the all-equity firm is $[x(T) - I]$ and the value of the trading firm at date $t$ is the present value of $x(T)$ minus the present value of taxes to be paid and the cost of production $I$. As the firm's income at date $T$ net of taxes is a concave function of its pre-tax income, Jensen's Inequality implies that the all-equity trading firm's expected income net of taxes is smaller than the after-tax value of its pre-tax expected income. Consequently, the value of the all-equity firm is maximized if it sells forward its income to obtain its expected income for sure. As the marginal tax rate for individuals is fixed, the forward price of the real exchange rate is equal to its expected value, so that the all-equity trading firm can achieve its first-best hedging policy if a forward contract of the appropriate maturity is available or can be constructed through a self-financing portfolio strategy.

Suppose now that the trading firm can achieve its first-best hedging policy. In this case, the trading firm can further increase its value by selling debt up to the point at which its taxable income reaches $Y_{m}$, where $Y_{m}$ is defined implicitly by the relation $\tau_{m} = \tau(Y_{m})$. For the firm's taxable income to reach $Y_{m}$, it may have to sell debt whose proceeds exceed $I$. In this case, we assume that the firm pays a dividend equal to the proceeds of the debt issued in excess of $I$ and the dividend is taxed at the rate $\tau_{m}$. It does not pay for the trading firm to issue more debt, because the tax shield of an additional dollar of debt would be smaller than the taxes the marginal investor would have to pay on the interest payments of the additional debt. When the firm follows its first-best hedging policy, bankruptcy costs are irrelevant because bankruptcy never occurs.

At this point, one might be tempted to argue that by hedging less, the trading firm can obtain a bigger tax shield of debt. The argument goes as follows. As the probability of bankruptcy increases, the yield on the firm's debt increases and hence so does the tax shield in states of the world in which the firm is bankrupt. To show that we identified the firm's optimal debt and hedging policies, we consider only the case in which bankruptcy costs are equal to zero, as bankruptcy costs reinforce our argument. In this case, increasing the variance of the trading firm's
before tax and before interest payments income while maintaining its mean constant can only decrease the firm's value. To understand this, note that, after the increase in variance, the firm's tax rate will be higher than \( \tau_m \) in some states of the world and lower than \( \tau_m \) in other states of the world. It follows that the total taxes paid on the firm's income would be reduced if individuals and the trading firm would trade securities so that the firm's income is increased when \( \tau < \tau_m \) and decreased when \( \tau > \tau_m \). The firm's optimal hedging policy enables it to equate its marginal tax rate to the individuals' marginal tax rate and hence to minimize the firm's and the individuals' joint tax liability in each state of the world.

In this setting, the trading firm can follow its optimal hedging policy by selling the proceeds from its foreign sale on the forward market. In the absence of a traded real exchange rate forward contract that matures when the firm sells its production, the trading firm can pursue a dynamic hedging policy. The firm can follow its optimal hedging policy provided that it can construct a self-financing portfolio that pays \( x(T) \) at time \( T \). If the real exchange rate follows a random walk, the construction of such a portfolio is straightforward. That is because the rate of return in terms of the foreign consumption good of a bond which pays one unit of the foreign consumption good at maturity is non-stochastic, so that the value of that bond in the domestic country is a linear function of the real exchange rate. Consequently, a sufficient condition for the trading firm to be able to implement its optimal financing and hedging policies is that a default-free discount bond which pays one unit of the foreign consumption good at maturity is traded at all times during the life of the firm. If the real exchange rate follows a mean-reverting process, the rate of return in the domestic country on a bond denominated in the foreign consumption good is stochastic. However, the price of the bond will be solely a function of the real exchange rate and time, so that a dynamic hedging policy can be implemented by the firm if the real exchange rate has a continuous sample path as long as such a bond exists. If the real exchange rate follows a mean-reverting process with lognormally distributed jumps, the existence during the life of the firm of a traded discount bond that pays one unit of the foreign consumption good at maturity is not sufficient to enable the firm to pursue its first-best hedging policy. This is because, since the changes in the real exchange rate can be large, changes in the value of the firm are not proportional to changes in the real exchange rate in the limit of continuous-time. Consequently, in this case, a sufficient condition for the firm to be able to pursue its first-best hedging policy is that it can take a position in a discount bond or a forward contract whose payoff at date \( T \) is proportional to the real exchange rate at that date. This means that, when the real exchange rate follows a mean-reverting process with jumps, the firm's hedging policy may be constrained by the investment opportunity set unless a discount bond or a forward contract that matures exactly when the firm sells its production and whose payoff at that date is proportional to the real exchange rate is available. The value of the firm, for given pre-tax cash flows, is lower when it cannot follow its first-best hedging policy. Further, an increase in the volatility of the real exchange rate has...
no effect on the value of the firm when it is able to follow its first-best hedging policy, but decreases it otherwise.

We now turn to the case in which the firm has a project of the type described in Section 4 that involves an option to export. At date $T$, the pre-tax value of the firm is a linear function of the real exchange rate if at $t^*$ the firm decided to export and is a constant otherwise. This means that, if the firm exports, the tax schedule implies again that the after-tax value of the firm is a concave function of its pre-tax value, so that the firm's first-best hedging policy is the same as in the case in which the firm does not have an option to export. Now, however, the pre-tax value of the firm is always a non-linear function of the real exchange rate before date $t^*$, i.e., before the date at which the option expires. This means that the firm must use real exchange rate options to hedge or be able to replicate dynamically the payoffs of such options. In fact, the firm wants to hold a portfolio whose payoff at date $t^*$ is equivalent to a real exchange rate option that matures at that date. If the firm exercises its option to export at date $t^*$, it must then hold a portfolio whose payoff at date $T$ is proportional to the real exchange rate at that date. If the real exchange rate follows a diffusion process, a large number of financial instruments can be used by the firm to achieve its first-best hedging policy. Any portfolio whose return is perfectly correlated locally with the rate of growth of the real exchange rate enables the firm to replicate the payoff of a real exchange rate option dynamically in this case. However, if the path of the real exchange is not continuous, a real exchange rate option's payoff cannot be replicated dynamically by trading in a safe bond and an asset whose return is perfectly correlated locally with the rate of growth of the real exchange rate. The reason for this is the same as the reason for why the payoff on a option on common stock cannot be replicated by trading in the stock and a default-free bond if the path of the common stock is not continuous. This is because, in this case, the rate of return of the option is not a linear function of the rate of growth of the real exchange rate in the limit of continuous-time. Consequently, if the real exchange rate dynamics incorporate a jump process, the firm has to hold a position in a real exchange rate option that matures at date $t^*$ to achieve its first-best hedging policy. In the absence of such an option, the value of the firm is lower. This does not mean, however, that the value of the firm falls with an increase in real exchange rate volatility, since the value of the firm's option to export is an increasing function of the volatility of the real exchange rate.

This section identifies optimal hedging and financing policies for the trading firm. The existence of such optimal policies results from the existence of progressive taxes for the firm with a maximum tax rate in excess of the marginal tax rate of the marginal investor. Although our analysis ignores the non-debt tax shields that play a crucial role in the analyses of DeAngelo/Masulis [4] and Kim [15], our results are the same irrespective of whether the progressivity of corporate taxation results from the tax schedule or is brought about by the existence of non-debt tax shields. If marginal tax rates are constant and the marginal tax rate of the marginal investor is in excess of the highest corporate tax rate, an interior solution for the
firms' capital structure does not exist, as discussed by Talmor//Haugen/Barnea [24]. However, in this case, as long as corporate taxes are progressive, the value of the firm is increased by an active hedging policy. This conclusion holds even if the marginal tax rate of the marginal investor is state-dependent.

Although our model leads to the conclusion that a firm's financing and hedging policies affect its value, this conclusion holds even if $x$ is the price of peanuts instead of the real exchange rate. It also holds if $x$ corresponds to a measure of the firm's revenue. Hence, the optimal policies identified here apply in a variety of cases and provide a raison d'être for many financial instruments. While earlier papers pointed out that progressive taxes lead to the existence of optimal hedging policies, they ignored personal taxes and did not show how the value of the firm is tied to the type of financial instruments available. In this paper, the ability of firms to pursue their first-best hedging policy depends crucially on the extent to which they can form self-financing portfolios whose payoffs are known functions of the real exchange rate. The paucity of financial instruments whose payoffs are known functions of the real exchange rate would suggest that it may be difficult for firms to pursue their first-best hedging policy. However, to the extent that nominal exchange rates are highly correlated with real exchange rates, financial instruments whose payoffs are known functions of the nominal exchange rate may be good substitutes for financial instruments whose payoffs depend explicitly on the real exchange rate.

Section 6. Concluding Remarks

This paper provides an analysis of the effect of alternative assumptions about real exchange rate dynamics on the trading firm's capital budgeting decisions and its financing policies. The analysis assumes that the trading firm undertakes a simple project. Although it can be argued that most projects can be viewed as portfolios of simple projects of the type studied here, it would be interesting to extend the present analysis to more complicated projects whose costs also depend on the real exchange rate. Further, many of our results could be applied to projects whose value does not depend on the real exchange rate but, instead, depends on some other hedgeable stochastic variable. Finally, it would be interesting to see how our results are changed if the tax code is made more realistic.
Notes

1. The real exchange rate is the price of the foreign currency in real terms; it is formally defined as \( e\pi^*/\pi \), where \( \pi^* \) is the foreign price level, \( \pi \) is the domestic price level and \( e \) is the domestic currency price of one unit of foreign currency.

2. See, for instance, Dornbusch [5] and Roll [21] for discussions of these alternative views.

3. Burt/Kaen/Booth [3], Westerfield [25] and Rogalski/Vinso [20] found that rates of change of spot exchange rates are not normally distributed. However, Hsieh [12] shows that such results might be caused by the fact that the variance of the rate of change of exchange rates changes stochastically over time. While these studies focus on nominal exchange rates, the real exchange rate is highly correlated with the nominal exchange rate.

4. This paper is most closely related to Dumas [7] who provides, in a two-period model without taxes, a systematic exploration of the corporate finance issues faced by the trading firm.

5. See Huizinga [13] for a discussion of the literature and evidence that, over long periods of time, one cannot reject the hypothesis that a substantial fraction of exchange rate changes is temporary.


7. Jones/Purvis [14] and others have built models using the assumption that the only traded goods are intermediate goods and argued its merits. For convenience, we call the consumption good available for sale abroad the foreign consumption good, even though it differs from the consumption good available for sale at home only by its location.

8. See, for instance, Mussa [18].

9. The model is closely related to the one used in Stulz [23]. Consequently, that model can be used to study how our conclusions are changed if individuals are risk-averse. Note, however, that there is no reason to assume that risk is priced differently across different real exchange rate regimes.

10. For simplicity, we assume that if the value of the firm is the same irrespective of which project it adopts, it chooses to produce for the domestic market.

11. See, for instance, Garman/Kohlhagen [9] or Grabbe [10]. Their result is more general because here, using the consumption good as the numeraire, real interest rates are the same at home and abroad, since the real exchange rate follows a martingale.

12. Note that if the option is written on common stock instead, \( r_n \) is equal to \(-\lambda k + r + r\gamma/s\).

13. For excellent discussions of the issues involved in the choice of the interest first or of the principal first doctrine, see Talmor/Haugen/Barnea [24] and Park/Williams [19].

14. Let \( V(Y) \) be the firm’s income net of taxes. Differentiating \( V(2Y) \) twice, we have \( V''(Y) = -r^*Y - 2\gamma = 0 \). By construction, the firm cannot carry losses back or forward. If we allow the firm to use losses to reduce taxes in other periods, \( V(Y) \) can still be a concave function of \( Y \).

15. This assumption is reasonable given the 1986 Tax Reform Act. For 1988, the highest income tax rate for individuals is 28% while the highest income tax rate for corporations is 34% for most firms and 39% for some firms.

16. This argument is used in Smith/Stulz [22]. See also Green/Talmor [11]. However, these authors do not take into account personal taxes.
17. This follows from the fact that the expected real return in terms of the domestic consumption good of a bond which has a certain payoff in terms of the foreign consumption good must be equal to the domestic real rate of interest.

18. Formally, if \( V(x,t) \) is the value of the firm at date \( t \) in the absence of taxes, we have:

\[
V(xH,t) - V(x,t) = V_x(x,t)(xH - x),
\]

where \( V_x(x,t) \) is the partial derivative of \( V(x,t) \) with respect to \( x \).

19. See Merton [17].

20. In particular, our results also apply to Brennan/Schwartz [2].

References


