

## THE DEMAND FOR FOREIGN BONDS

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This paper derives the demand for foreign bonds in a simple general equilibrium model in which the exchange rate is perfectly correlated with the terms of trade. A necessary condition for the demand for foreign bonds to be an increasing function of the domestic currency value of imports is derived. Earlier work shows that the demand for foreign bonds is an increasing function of imports if the domestic investor's degree of relative risk tolerance is smaller than one. The present paper shows that the demand for foreign bonds can be a decreasing function of imports if the degree of relative risk tolerance is smaller than one when the consumption expenditure elasticity of imports exceeds one.

### 1. Introduction

There is a growing literature in international economics which uses the principle of expected utility maximization to derive asset demands in open economies.<sup>1</sup> The importance of this literature derives from the fact that asset demands play a crucial role in explaining exchange rate changes in the portfolio approach to exchange rate determination.<sup>2</sup> For instance, an unanticipated current account surplus increases domestic relative wealth. Proponents of the portfolio approach to exchange rate determination have argued that, if domestic investors have a greater propensity to hold domestic bonds rather than foreign bonds, the redistribution of wealth due to the current account surplus leads to an excess demand for domestic bonds. Equilibrium will be restored on financial markets only if the domestic currency appreciates and/or the domestic interest rate decreases relatively to the foreign interest rate. The redistribution of world wealth caused by an unanticipated current account surplus has different effects if domestic

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<sup>1</sup>For instance, see Solnik (1973), Kouri (1976), Kouri and Macedo (1978), Dornbusch (1980b), Macedo (1980a, 1980b) and Krugman (1981) for models which are related to the model developed in this paper.

<sup>2</sup>For references, see for instance Branson et al. (1977), Kouri and Macedo (1978), Dooley and Isard (1979), Obstfeld (1979) and Dornbusch (1980a, 1980b).

investors do not have a greater propensity to hold domestic currency assets than foreign currency assets.

This paper presents a simple general equilibrium model which offers a precise statement of the determinants of the demand for foreign bonds. Since it is assumed that each country produces only one commodity, which it uses as a numeraire, the model makes an important assumption which has also been made in models developed recently by Macedo (1980a) and Krugman (1981). However, the major result of the paper, which is the condition which must be satisfied for the domestic demand for foreign bonds to be an increasing function of the value of domestic imports, is substantially different from a result obtained by Macedo and Krugman. They found that the demand for foreign bonds is an increasing function of the value of imports if the domestic investor's coefficient of relative risk tolerance  $T^R$  is smaller than one. It is shown in this paper that the demand for foreign bonds is an increasing function of the value of imports if  $\varepsilon T^R$  is smaller than one, where  $\varepsilon$  is the domestic consumption expenditure elasticity of imports (i.e. the ratio of the marginal and average propensities to import).<sup>3</sup> An immediate implication of this result is that the demand for foreign bonds which arises from the fact that the investor consumes foreign goods is negative if  $\varepsilon T^R$  is larger than one.

This crucial difference between the papers of Macedo and Krugman and this paper arises from the fact that in the present paper no significant restriction is put on the utility function of the domestic investor, whereas both Macedo and Krugman constrain the utility function to belong to the class of utility functions which exhibit constant expenditure shares. If one accepts that a reasonable guess for the range of plausible values of  $T^R$  is between one-half and one,<sup>4</sup> it is possible for the model developed here to offer predictions which contradict the predictions obtained from the work of Macedo and Krugman whenever  $\varepsilon$  exceeds one.

An example may help to understand why the difference between the results of this paper and the results of Macedo and Krugman matters. Dornbusch (1980a) argues that bonds denominated in marks are an attractive asset to hold in a portfolio diversified across currencies. To make his point, he shows that if an investor chooses to hold a minimum-variance portfolio constituted

<sup>3</sup>Rigorously, the result is that the demand for foreign bonds increases as a function of imports if:

$$\frac{\partial C_e}{\partial M} + (1 - T^R \varepsilon) > 0,$$

where  $C_e$  is the partial derivative of the consumption expenditure function with respect to the logarithm of the exchange rate. In most of this paper it is assumed that  $\partial C_e / \partial M$  is negligible, which occurs if expected returns are not constant only if  $T^R$  is constant. Note, however, that if  $\varepsilon \neq 1$ ,  $T^R$  cannot be the same for all levels of consumption expenditures, which makes it impossible to assume formally that  $T^R$  is constant. See Stiglitz (1969).

<sup>4</sup>Krugman (1981) states that one-half is a widely accepted value. Arrow (1965) argues that  $T^R$  hovers around one.

of bonds denominated in marks and in dollars, about half of this portfolio is invested in mark bonds. He then argues that diversification into mark denominated assets may help to understand why the mark appreciated in the 1970s. Dornbusch noticed, however, that differences in consumption patterns might make mark denominated assets less attractive for U.S. investors. With the results of this paper, the demand for mark denominated bonds which arises from the fact that U.S. investors import consumption goods from Germany is negative if  $\varepsilon T^R > 1$ . If  $\varepsilon T^R$  is large enough, U.S. investors want to be short in marks and one has to find some other explanation for the appreciation of the mark. Furthermore,  $\varepsilon T^R$  could be larger than one even though  $T^R$  is smaller than one.

The general equilibrium nature of the model ensures that the dynamics of the endogenous variables assumed by investors when they solve their optimal consumption and portfolio strategies are indeed the true dynamics for these variables. Furthermore, it makes it possible to study how robust the main result is to the introduction of more realistic features in the model. It is shown that the main results of the paper still hold if the state vector is expanded to include additional state variables, provided that if the dynamics of the additional state variables are correlated with exchange rate changes, there exist assets whose returns are perfectly correlated with the dynamics of these state variables.

The plan of the paper is as follows. Section 2 presents the model and derives the asset demands for a representative domestic investor. Section 3 analyzes the demand for foreign bonds and discusses the conditions under which the results of the paper hold if the vector of state variables is expanded. Section 4 offers some concluding remarks.

## 2. The model

### 2.1. Asset returns

Without loss of generality, it is assumed that there are only two countries, the domestic country and the foreign country. Each country produces only one good which it uses as a numeraire. With these assumptions, the exchange rate  $e$  is the domestic price of one unit of the foreign commodity. It is assumed that the state of the world can be represented by an  $S \times 1$  vector of state variables,  $S$ , and that the dynamics of the state variables follow Itô processes.<sup>5</sup> The exchange rate is a function of the state variables<sup>6</sup> and its

<sup>5</sup>The properties of Itô processes and of the stochastic differential equations they follow are given in Merton (1971). See also Arnold (1974) and other references given in Breeden (1979). Fischer (1975) has an elegant introduction to the methodology used in this paper.

<sup>6</sup>Cox, Ingersoll and Ross (1978) explain how one can solve analytically for the value of endogenous variables and develop the general equilibrium methodology used here. In this paper, however, the function which yields the exchange rate is left unsolved. To provide an analytical

dynamics can be written as:

$$\frac{de}{e} = \mu_e dt + \sigma_e dZ_e, \quad (1)$$

where  $\mu_e$  is the instantaneous expected rate of change of the exchange rate,  $\sigma_e$  is the instantaneous standard deviation of the rate of change of the exchange rate and  $dZ_e$  is the increment of a Wiener process.

For the sake of simplicity, it is assumed that there is only one domestic investor. He faces no barriers to international investment and can invest in shares of domestic and/or foreign firms. It is assumed that all firms in a country use the same constant returns to scale technology so that investments in each firm of that country have exactly the same returns. Firms use as input the commodity they produce. There are no barriers to entry in an industry. With these assumptions, perfect competition obtains and the value of the domestic (foreign) industry in domestic (foreign) currency is equal to the existing stock of the domestic (foreign) commodity, written  $K$  ( $K^*$ ). For the state variables to follow Itô processes, it is necessary that  $K$  and  $K^*$  follow Itô processes, which implies that the instantaneous return on an investment in the domestic industry can be written as:

$$\frac{dX}{K} = \mu_K dt + \sigma_K dZ_K, \quad (2)$$

where  $\mu_K$  is equal to  $(1/dt)E(dX/K)$ ,  $\sigma_K^2$  is equal to  $(1/dt)\text{var}(dX/K)$  and  $dZ_K$  is the increment of a Wiener process.  $dX$  exceeds  $dK$ , i.e. the change in the value of the domestic industry, by the number of units of the domestic commodity consumed per unit of time. The foreign currency instantaneous rate of return on an investment in the foreign industry is analogously defined as:

$$\frac{dX^*}{K^*} = \mu_{K^*} dt + \sigma_{K^*} dZ_{K^*}. \quad (3)$$

Using Itô's Lemma, the domestic currency return on an investment in the foreign industry is given by:

$$\frac{d(eX^*)}{eK^*} = (\mu_{K^*} + \mu_e + \sigma_{eK^*})dt + \sigma_e dZ_e + \sigma_{K^*} dZ_{K^*}, \quad (4)$$

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solution, it would be necessary to make assumptions about the utility function of the investors which would make it impossible to develop the main result of the paper. It is assumed that the exchange rate function is sufficiently differentiable so that Itô's Lemma can be applied.

where  $\sigma_{eK^*}$  is the instantaneous covariance per unit of time between the foreign currency returns and the rate of change of the exchange rate. It is assumed that  $\mu_K$ ,  $\mu_{K^*}$ ,  $\sigma_K$  and  $\sigma_{K^*}$  are intertemporal constants.

The investor can also invest in a domestic currency bond whose instantaneous rate of return at home,  $R$ , is non-stochastic. Finally, the investor can invest in a bond whose instantaneous return in foreign currency,  $R^*$ , is non-stochastic. Both  $R$  and  $R^*$  are functions of the state variables. The instantaneous return on a foreign currency bond in the domestic currency is given by:

$$\frac{d(eB^*)}{eB^*} = (R^* + \mu_e)dt + \sigma_e dZ_e, \quad (5)$$

where  $B^*$  is the foreign currency price of the bond.

## 2.2. Asset demands

It is assumed that the domestic investor maximizes a lifetime expected utility function which is von Neuman–Morgenstern, time-additive and depends only on the consumption of the two commodities and time. The expected utility function of the investor is given by:

$$E_t \left\{ \int_t^\infty e^{-\rho s} U(C_1(s), C_2(s)) ds \right\}, \quad (6)$$

where  $E_t$  is the expectation operator conditional on information available at time  $t$ .  $C_1(s)$  is the consumption rate of the domestic good and  $C_2(s)$  is the consumption rate of the foreign good. The domestic investor maximizes (6) subject to a stock budget constraint given by:

$$n + n^* + b + b^* = 1, \quad (7)$$

where  $n$  ( $n^*$ ) is the share of the investor's wealth  $W$  invested in the domestic (foreign) industry and  $b$  ( $b^*$ ) is the share of the investor's wealth invested in the domestic (foreign) default-free bond. There are no restrictions on borrowing at home or abroad and there are no restrictions on short-sales.

The domestic investor must also satisfy a flow budget constraint. Making use of the stock budget constraint and of a compact notation, the flow budget constraint can be written as:

$$dW = \sum_{i=1}^3 w_i \frac{dH_i}{H_i} W + RWdt - C_1 dt - eC_2 dt, \quad (8)$$

where

$$\frac{dH_1}{H_1} = R^*dt + \frac{de}{e} - Rdt, \quad (9)$$

$$\frac{dH_2}{H_2} = \frac{dX}{K} - Rdt, \quad (10)$$

$$\frac{dH_3}{H_3} = \frac{dX^*}{K^*} + \sigma_{eK^*}dt - R^*dt \quad (11)$$

and  $w_1 = b^* + n^*$ ,  $w_2 = n$  and  $w_3 = n^*$ . From now on,  $dH_i/H_i$  is defined as the excess return on the  $i$ th risky asset. The instantaneous excess return on a risky asset is the instantaneous return on that asset in excess of the return on the safe bond available in the home country of that asset. Note, however, that the foreign bond is risky only in the domestic country, which explains that its excess return is defined as its return in domestic currency in excess of the domestic rate of interest. The transformation of returns into excess returns is useful since it eliminates the dependence of the return on investments in the foreign industry on the stochastic part of the exchange rate dynamics and therefore makes the foreign bond the only asset whose return depends on the stochastic part of exchange rate dynamics. Notice also that  $w_2$  is the fraction of the investor's wealth invested in the foreign industry when the return on this investment is hedged against exchange rate risks.

It is now assumed that there is only one foreign investor and that he faces the same consumption and investment opportunity sets as the domestic investor. The foreign investor's lifetime expected utility function satisfies the same assumptions as the domestic investor's lifetime expected utility function. In order for the model to be a general equilibrium model, i.e. a model in which the asset and commodity price dynamics implied by the investors' optimizing consumption and portfolio choices are the asset and price dynamics they assume to hold when making their consumption and portfolio choices, it is necessary that the vector of state variables incorporates the available stocks of commodities, i.e.  $K$  and  $K^*$ , and the aggregate wealth of the domestic investor, i.e.  $W$ , and of the foreign investor, i.e.  $W^*$ . By construction, no other state variable can affect price dynamics. If at two different dates  $K$ ,  $K^*$ ,  $W$  and  $W^*$  have the same values, it follows that all endogenous variables also have the same values. Given the values of  $K$ ,  $K^*$ ,  $W$  and  $W^*$ , it is possible to solve for the exchange rate,  $e$ . Since the function  $e(S)$  can be inverted, the domestic investor can solve his optimization problem using as state variables  $e$ ,  $K$ ,  $K^*$  and  $W$ . In this case, the vector  $S$  can be chosen to be a  $3 \times 1$  vector which includes the logarithms of  $e$ ,  $K$  and  $K^*$ , which are all observable variables for the investor.

Define  $\mu^H$  to be the vector of instantaneous expected excess returns,  $V_{aa}$  to be the  $3 \times 3$  variance-covariance matrix of instantaneous excess returns and  $V_{aS}$  to be the  $3 \times 3$  covariance matrix of instantaneous excess returns and changes in state variables. Finally, let  $w$  be the  $3 \times 1$  vector of investment proportions in the risky assets. With this notation, the asset demands are given by:

$$w = \left( \frac{C}{C_W W} \right) T^R V_{aa}^{-1} \mu^H + V_{aa}^{-1} V_{aS} \left\{ \frac{-C_S}{C_W W} + \left[ \frac{1}{C_W W} (1 - T^R \varepsilon) M \right] \right\}, \quad (12)$$

where  $C$  is equal to consumption expenditures, i.e.  $C = C_1 + eC_2$ , which are a function of the wealth of the investor and the state variables, i.e.  $C = C(W, S)$ .  $T^R$  is the relative risk tolerance of the investor's indirect utility function of consumption expenditures,  $M$  is the domestic currency value of imports and  $\varepsilon$  is the consumption expenditure elasticity of imports.<sup>7</sup>

### 2.3. Asset demands and market completeness

Notice now that the excess return on the foreign bond is perfectly correlated with changes in the exchange rate and that the excess return on an investment in the domestic (foreign) industry is perfectly correlated with changes in  $K$  ( $K^*$ ). It follows that the domestic investor can hedge perfectly against unanticipated changes in state variables. The economy considered here has the feature that markets are complete in an Arrow-Debreu sense. Since markets are complete and since there are as many state variables as there are assets, it can be shown that  $V_{aa} = V_{aS}$  and asset demands can be rewritten as:

$$w = \left( \frac{CT^R}{C_W W} \right) V_{aa}^{-1} \mu^H + \frac{1}{C_W W} \left\{ -C_S + \begin{bmatrix} (1 - T^R \varepsilon) M \\ 0 \\ 0 \end{bmatrix} \right\} \quad (13)$$

Eq. (13) can be interpreted as a weighted sum of four column vectors, such that the vectors do not depend on tastes and preferences whereas the weights do. The column vectors, appropriately scaled, can each be interpreted as portfolios. The first mutual fund portfolio is a mean-variance efficient portfolio (i.e. there is no portfolio which dominates this portfolio by having a higher mean for the same variance or a lower variance for the same mean)

<sup>7</sup>Stulz (1980) provides a general version of the present model.

given by the  $3 \times 1$  vector  $w^m = [(V_{aa}^{-1} \mu^H)' I]^{-1} V_{aa}^{-1} \mu^H$ , where  $I$  is a  $3 \times 1$  vector of ones. The other three mutual fund portfolios, whose investments in risky assets are given respectively by the three column vectors of a  $3 \times 3$  identity matrix, are used by investors to hedge against unanticipated changes in state variables. For instance, a portfolio of shares in the foreign industry allows the investor to hedge against unanticipated changes in the existing stock of the foreign commodity.

### 3. Economic implications of the model

#### 3.1. *The demand for foreign bonds*<sup>8</sup>

The domestic investor's holdings of foreign bonds can be decomposed into two parts. First, the investor has a speculative demand for foreign bonds which does not depend on his consumption preferences and is given by the holdings in foreign bonds of the mean-variance efficient portfolio  $w^m$  times his wealth  $W$ , i.e.  $w_1^m W$ . An investor who holds a portfolio of investments in the domestic and foreign industries chooses to hold foreign bonds if these bonds make it possible to construct a portfolio which dominates, in mean-variance space, the mean-variance efficient portfolio of investments in industries. The absolute value of these holdings of the foreign bond increases with the relative risk tolerance of the investor and does not depend on the consumption preferences of the investor. The investor makes a speculative investment in foreign bonds whenever  $R^* + \mu_e - R$ , i.e. the expected excess return on foreign bonds, is different from zero, and/or whenever the exchange rate is correlated with the domestic or the foreign output rate (i.e. the first row of  $V_{aa}$  has non-zero off-diagonal elements). Second, the investor holds foreign bonds because the return on these bonds is correlated with the changes in a state variable, i.e. the exchange rate, whose value affects his lifetime expected utility.

The demand for foreign bonds, explained by the fact that the exchange rate is a state variable, is given by:

$$w_1^H W = -\frac{C_e}{C_W} + \frac{(1 - T^R \epsilon) M}{C_W}, \quad (14)$$

<sup>8</sup>In the following  $w_1 W$  is called the demand for foreign bonds. More rigorously,  $b^* W$  corresponds to the investor's holdings of foreign bonds and  $b^*$  is equal to  $w_1 - n^*$ , where  $n^* W$  is equal to the investor's holdings of shares in the foreign industry. However, an increase in  $b^*$  due to a decrease in  $n^*$  does not correspond to a change in the investor's net holdings of foreign assets or net wealth and, consequently,  $b^* W$  is not useful to address the issues raised by the portfolio approach to exchange rate determination.  $w_1 W$  is the investor's demand of assets whose return is perfectly correlated with changes in the exchange rate.  $w_1 W$  differs from  $b^* W$  because the investor hedges his holdings of shares of foreign industries against unanticipated changes in exchange rates.

where  $C_e$  is the partial derivative of the consumption function with respect to the logarithm of the exchange rate. The investor's holdings of foreign bonds given by (14) are such that an unanticipated change in the exchange rate affects the value of  $w_1^H W$  so as to leave the marginal utility of consumption expenditures constant.

An unanticipated change in the exchange rate has two distinct effects on the marginal utility of consumption expenditures. First, for a constant amount of consumption expenditures, a change in the exchange rate affects the marginal utility of consumption expenditures because it affects the quantities of commodities the investor can consume for these expenditures. Second, a change in the exchange rate, for a given value of the investor's wealth, affects the amount of consumption expenditures itself, as the investor rearranges his lifetime consumption plans. In what follows the first effect is called the price-level effect, whereas the second effect is called the intertemporal substitution effect. The price-level effect is discussed in the next section.

The intertemporal substitution effect accounts for a demand of foreign bonds given by  $C_e/C_W$ . If the indirect utility function of consumption expenditures exhibits constant relative risk aversion,  $C_e=0$ . For general utility functions investors hedge against unanticipated changes in the investment opportunity set, i.e. asset prices and their joint distribution at future dates, and against unanticipated changes in the consumption opportunity set, i.e. the current price and the distribution of future prices of the foreign commodity. A change in the exchange rate is likely to be associated with changes in the investment and consumption opportunity sets, which explains that foreign bonds can be used to hedge against unanticipated changes in the investment and consumption opportunity sets. Whereas it is possible in principle to solve analytically for  $C_e$ , no analytical solution is known for this type of problem if  $C_e \neq 0$  and  $\varepsilon \neq 1$ .

### 3.2. *The price-level effect*

An increase in the exchange rate, *ceteris paribus*, increases the cost in domestic currency of the basket of commodities the domestic investor consumes if the foreign commodity enters his consumption. The larger the ratio of imports to consumption expenditures, i.e.  $M/C$ , the larger the change in wealth required to maintain the marginal utility of consumption expenditures constant if an unanticipated change in the exchange rate occurs. An investor who has no risk tolerance, i.e.  $T^R=0$ , wants his wealth to change so that he can still consume the basket of commodities he had planned to consume before the unanticipated change in the exchange rate. Such an investor owns foreign bonds for an amount equal to  $M/C_W$  and invests the remainder of his wealth in domestic default-free bonds. In this case, an

unanticipated change in the exchange rate given by  $\Delta e$  changes the value of the investor's holdings of foreign bonds by  $(C_e/C_w)\Delta e$ . The change in consumption expenditures explained by a change in wealth equal to  $(C_e/C_w)\Delta e$  is given by  $C_w(C_e/C_w)\Delta e = C_e\Delta e$ . It follows that after an unanticipated change in the exchange rate given by  $\Delta e$  the investor is still able to consume the basket of commodities he would have consumed without that change.

The demand for foreign bonds explained by the price-level effect is given by  $(1 - T^R \varepsilon)M/C_w$ . From the above discussion,  $M/C_w$  corresponds to the investment in the foreign bond required to hedge the investor's consumption of the foreign commodity exactly against unanticipated changes in the exchange rate. The second part of  $(1 - T^R \varepsilon)M/C_w$ , i.e.  $-T^R \varepsilon M/C_w$ , is explained by the fact that an investor who is willing to bear some risk to increase his expected consumption at future dates is also willing to bear some commodity price uncertainty if, by doing so, he can increase his expected consumption at future dates. The higher the investor's risk tolerance, the more willing he is to enter gambles which involve price uncertainty and have a positive expected real excess return.

A foreign bond is equal to a gamble which involves price uncertainty. If  $\mu_1^H = 0$ , the expected excess return of the gamble in home currency is equal to zero. To find the expected real excess return of the gamble, it is necessary to use a price index whose value depends on how the investor spends an unanticipated additional dollar of returns. Let  $m$  be the investor's marginal propensity to spend on the foreign good, i.e.  $m = \varepsilon M/C$ . Since the price of the domestic good is constant,  $dP^m/P^m = m(de/e)$  measures the rate of change of the price  $P^m$  of the basket of commodities the investor buys with an unanticipated dollar of returns.

The expected real excess return of the foreign bond must be obtained by using the price index  $P^m$ , since that price index measures the purchasing power of unanticipated returns. Using Itô's Lemma, it follows that if  $\mu_1^H = 0$ , the expected real excess return on the foreign bond per unit of time is equal to  $(1/dt)\text{cov}(de/e, dP^m/P^m) = -m\sigma_e^2$ . If  $\mu_1^H = 0$ , the expected real excess return on the foreign bond is negative and therefore the investor goes short in the foreign bond, because a negative investment in the foreign bond has a positive expected real excess return. This result follows from the fact that the real value of the foreign bond is a convex function of the price index  $P^m$  and that Jensen's Inequality applies.<sup>9</sup> The crucial fact in the argument, however, is that the appropriate instantaneous rate of change of the price level is given by  $m(de/e)$  rather than by  $(M/C)(de/e)$ , which is the appropriate instantaneous rate of change of the price level if the investor has a utility function which exhibits constant expenditure shares.

<sup>9</sup>Krugman (1981) provides a detailed discussion of how Jensen's Inequality affects asset demands.

Going short in the foreign bond is taking a gamble whose expected real excess return is positive and is an increasing function of the marginal propensity to spend on the imported good. The investor's demand for the gamble is an increasing function of his risk tolerance and of the expected return of the gamble. To understand this, it is useful to notice that  $(T^R \varepsilon M)/C_W = T^A m/C_W$ , where  $T^A$  is the absolute risk tolerance coefficient of the investor. Since the variance of the real excess return of the gamble is  $\sigma_e^2$ , the ratio of the expected real excess return of the gamble and its variance does not depend on the variance of the real excess return of the gamble. This last fact explains why the investor does not consider the variance of the real excess return of the gamble when he decides how much he wants to participate in the gamble.

It is now possible to summarize the price-level effect on the demand for foreign bonds. The investor can be viewed as first hedging his current consumption completely against unanticipated changes in the exchange rate. This yields holdings of foreign bonds given by  $M/C_W$ . Next, the investor can be viewed as taking a gamble on the exchange rate. If the investor spends on the foreign good at the margin, a gamble which has an expected excess return in dollars equal to zero but whose return is perfectly negatively correlated with changes in the exchange rate has a positive expected real excess return. As the individual takes this gamble, he takes an additional short position in the foreign bond which depends positively on his risk tolerance and the expected gain of the gamble. If the expected gain of the gamble is sufficiently big, which is equivalent to saying that the fraction of an increase in income spent on the imported good is sufficiently large, the expected gain of the gamble overwhelms the investor's desire to hedge some of his current consumption against unanticipated changes in the exchange rate and  $w_1^H < 0$  if  $C_e \approx 0$ . Note that, for a given  $m$ , the smaller the fraction of current consumption expenditures spent on the foreign good, the more likely it is that  $w_1^H < 0$ , which explains that  $M/C$  and  $m$  have to be considered together to find the sign of  $w_1^H$  and consequently explains the role of the consumption expenditure elasticity to spend on the imported good.

### 3.3. *A special case*

For this example, it is assumed that there exists a closed convex set  $\Omega \subset R^4$  such that if the vector formed by  $W$ ,  $W^*$ ,  $K$  and  $K^*$  belongs to the set  $\Omega$ , investors have constant relative risk tolerance and  $C_e \approx 0$ . In this case, the holdings of foreign bonds explained by the fact that the domestic investor consumes the foreign commodity are proportional to:

$$w_1^H W \propto [1 - T^R \varepsilon] M,$$

where  $T^R$  is the relative risk tolerance of the domestic investor,  $\varepsilon$  is the

consumption expenditure elasticity of imports and  $M$  is the value of imports in domestic currency. It follows that holdings of foreign bonds, explained by the fact that the investor consumes the foreign good, are:

- (1) a decreasing function of the relative risk tolerance of domestic investors;
- (2) a decreasing function of the consumption expenditure elasticity of imports; and
- (3) an increasing function of imports if  $T^R \varepsilon < 1$  and a decreasing function otherwise.

Suppose that the coefficient of relative risk tolerance is equal to 1. In this case the investor's holdings of foreign bonds are an increasing function of imports only if  $\varepsilon < 1$  and are a decreasing function otherwise. Alternatively, suppose that  $T^R = \frac{1}{2}$ . In this case the investor's holdings of foreign bonds are an increasing function of imports only if  $\varepsilon < 2$ . It follows that  $T^R < 1$  is neither a necessary nor a sufficient condition for the investor's holdings of foreign bonds to be an increasing function of imports. In the economy presented here, a sufficient condition is that  $\partial C_e / \partial M = 0$  and  $T^R \varepsilon < 1$ . The results of Macedo and Krugman can be viewed as a special case of the results of this paper. Their results obtain if one assumes that relative risk aversion is constant and that  $C_e \equiv 0$ . Macedo and Krugman do not assume that  $C_e \equiv 0$  since they assume that the vector of expected excess returns is constant. In general equilibrium, if  $T^R$  is not identically equal to one and investors differ across countries, the vector of expected excess returns depends on the value of the state variables and is not constant through time.

### 3.4. An extension

It is possible to extend the model presented in this paper without changing the main results of the paper. To understand this, notice that the main results of this paper are obtained from the term  $w_1^H W$  in the demand for foreign bonds. This term is equal to the first row of a vector  $w^H W$  given by:

$$w^H W = \left( \frac{1}{C_w} \right) \mathbf{V}_{aa}^{-1} \mathbf{V}_{as} \left\{ -C_s + \begin{bmatrix} (1 - T^R \varepsilon) M \\ 0 \end{bmatrix} \right\}, \quad (15)$$

where  $0$  is a vector of zeroes. Since the number of assets and/or of state variables is changed, the demands for risky assets are affected. However, by inspection of (15),  $w_1^H W$  is not affected qualitatively if the first term of the column vector  $\mathbf{V}_{aa}^{-1} \mathbf{V}_{as} C_s$  is still equal to  $C_e$  as the number of assets and/or state variables is increased.

An example of an increase in the number of assets would be to increase the number of industries in each country, i.e. there would be many technologies available to produce the same good. Investors would buy shares

in the various industries. An example of an increase in the number of state variables would be to make  $\mu_K$ , i.e. the expected output rate, depend on some new state variable.

It follows from (15) that  $w_1^H W$  will have the same form as in this paper in two more general cases:

(1) unanticipated changes in the new state variables do not affect the investor's lifetime expected utility, or

(2) the first row of  $V_{aa}^{-1} V_{aS}$  has zeroes everywhere except for its first element, which is equal to one.

The second case holds either if markets are complete, in the sense that there exists a perfect hedge against unanticipated changes in each state variable, or if unanticipated changes in the new state variables are not correlated with unanticipated changes in the exchange rate. Notice, however, that the result that the demand for foreign bonds is an increasing function of imports only if  $T^R < 1$  is more robust, since it holds provided that  $\partial C_{S_i} / \partial M = 0$ ,  $\forall i$ , whenever the  $i$ th element of the first row of  $V_{aa}^{-1} V_{aS}$  is non-zero.

#### 4. Concluding remarks

In this paper the demand for foreign bonds has been derived in a general equilibrium framework and its properties have been studied. It has been shown, under suitable simplifying assumptions, that the demand for foreign bonds is an increasing function of imports if the product of the consumption expenditure elasticity of imports and the relative degree of risk tolerance of the investor is smaller than one. This result differs from other work in this area, for instance Krugman (1981) and Macedo (1980a), because it is not assumed here that investors have constant expenditure shares.

Several extensions of this paper could be of interest. For instance, the asset demands of section 3 could be used to find out whether there is a risk premium incorporated in the forward exchange rate and how it depends on consumption preferences. Additional simplifying assumptions could be made to obtain explicit solutions for the endogenous variables. Such an approach would make it possible to derive a closed-form equation for the exchange rate as a function of exogenous variables. Finally, it would be interesting to generalize the model to a world in which the exchange rate is imperfectly correlated with the terms of trade.

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