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ON THE LOGISTICS OF TALENT

by

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There has been deep and steadily growing concern in industry with the phenomenon of creativity. This is particularly vivid in our "talent intensive" industrial enterprises. The shortage of developed scientific and technical talent has become critical. An increasing number of people in positions of responsibility fear that we are moving toward a disaster. They feel that we must make every effort to find an imaginative course of action very soon and, hopefully, before the impending disaster overtakes us.

I believe there is convincing evidence that awakening of deep interest and deep commitment in the case of creative individuals usually occurs early in life. There have been some exceptions to this rule, both in life and in mythology.

As a society we have not been very imaginative in fostering young talent. We can make better use of much of the very large resources which are being expended in an attempt to energize members of the older generation whose attitudes have become stabilized. This last is done in a desperate effort to make up for the neglect of the very young who are also very talented.

Awareness in a young individual of some unusual opportunity for achievement has, at times, been very dramatic in its inception and in its effect. Ramanujan's case is a vivid example bordering on the miraculous. However, his experience, as incredible as it was, does exhibit the stages of development universal to other budding young minds and it provides a lesson not to be neglected.

I feel that the life of Ramanujan is not only a bright and heartwarming historical event, but it is also a vivid lesson providing a compelling motivation for treating young talent in a sensitive and imaginative manner.

The need for creativity in our science fiction society is so very great that we can no longer leave the needed awakening of talent to chance. The Nobel committee's citation in its latest economics award supports our view, and the latest Nobel prize winner in economics has been concerned with these and related issues.

My own involvement in the task of the early discovery and development of young talent goes back very far. This past summer was the thirtieth year of the formal existence of our special summer program designed for the able and eager youngsters (cf. Mathematical Scientist, 1978). Our program earned special commendations of the Mathematical Association of America (1986) and the American Association for the Advancement of Science (1987). I look forward to an opportunity of sharing with fellow members of this conference our trials and tribulations as we have worked at keeping alive our program and the ideas which it represents. Collaboration with our colleagues in Canberra, Heidelberg, and Bangalore has been deeply satisfying and I believe has profound implications.

Manifestations of creative scientific talent and mathematical talent in the very young vary widely,and our ability to read the early signs leaves much to be desired. Our ability to capture the attention of those for whom we have high hopes needs to be much improved. Providing the environment conducive to deep involvement of our young charges is a highly non-trivial task.

It is natural that the program which is intended as a memorial to Ramanujan should be intensely mathematical. We may ask if such a program can serve as an important (possibly critical!) influence upon the intellectual development of young people whose temperament and latent interests encourage them to consider scientific careers other than mathematics. I believe that it should do that and our experience has shown that it can. Let me explain.

It is more than a mere bon mot to say that science floats on a sea of mathematics. Thus, mathematical ideas should become a part of the intellectual equipment of every creative scientist. This is true not only of theoreticians but, because of the dramatic change in instrumentation, experimentalists as well. Can one remain initially within mathematics and develop vital components of scientific thinking such as the capacity for astute observation and also an eager curiosity leading to experimentation and courageous conjecturing? All of this must be present side by side with the searching outlook which compels a critical testing of conjectures through possible counterexamples and finally establishing a measure of security for surviving conjectures by a reasoned argument. Our experience has shown that with a proper choice of mathematical content, an appropriate treatment of this content and in the presence of esprit de corps induced by a warm "young scholars" community spirit, it is indeed possible to induce the quality of involvement which deepens mathematical thinking and, at the same time, increases the multiplicity of possible choices of life's work in science.

We may derive some comfort from the fact that of the twenty-seven (27) among the program alumni studying at MIT and Harvard at this time, nine (9) have majors in mathematics, four (4) have majors in physics, three

(3) in chemistry, three (3) in biology, five (5) in electrical engineering, two (2) in computer science, and one (1) has a major in economics.

A similarly wide distribution of interests is found among the mature alumni of our program. Of the four alumni on the faculty of Princeton, one is an economist (Sloan, 1986), one is a distinguished logician, one is a well known geophysicist, and one is a mathematician. A light country-wide scan of program alumni finds a young woman astrophysicist at Harvard-Smithsonian (Rossi High Energy Astronomy Award, 1986), and physicists at Livermore, University of Pennsylvania, and Cornell. Among the many accomplished mathematicians and computer scientists, one may mention one MacArthur award (Berkeley), one proof of Tate-Shafarevich conjecture (outstanding for 25 years), at least two Young Scientist Presidential Awards, numerous Sloan Fellowships, and among computer scientists one pioneer in symbolic manipulation by computers (IBM-Scratch Pad). At least four people concerned with the theoretical concerns of business and industry are on the faculty at the University of Illinois, The Ohio State University, and in Canada. The list can be much expanded.

We have tried to establish contact with an increasing number of our program alumni, both with those who are still very young as well as those who are established in their chosen professions. The quality of achievement and the diversity of interests of those of our alumni who have stayed in touch have been a source of deep satisfaction. I am glad to be able to report that IBM has become interested in our study and has offered to collaborate in a study which will blanket all thirty years of our program's existence. This is meant to be a detailed study of the pertinence to creativity of what we have been doing through the years in our work with the very young.

I believe that an effective way of outlining the basic considerations in our program design is to share the trials, tribulations, and soul-searching which beset us as we determined the direction of our effort. Since the history of our program has been discussed in detail in the Mathematical Scientist,[1] *let me outline it but briefly here.*

When in the summer of 1957, seven eager and bright high school youngsters approached us for guidance at the University of Notre Dame, Indiana, U.S.A. independently of each other, our summer program for secondary school teachers of mathematics was a going concern. With the memory of Sputnik still fresh in everyone's mind, our teacher-students were under great pressure to contribute to a reform of science education in their own schoolrooms. We on the faculty were expected to provide them with the mathematical insights and methods appropriate to the raising of a new generation of scientists.

The insertion of the seven youngsters into the secondary school teachers' group promised to test severely our choice of curriculum content as well as our teaching skills. Our young visitors rapidly put our misgivings to rest. They responded enthusiastically to every opportunity for involvement, grasped ideas which were traditionally considered inaccessible to the very young, and mastered relevant techniques so well as to perform outstandingly on the periodic tests taken together with over a hundred participating teacher-students.

It was not long before our high school participants began to relate their new intellectual experiences to their life ambitions varying from engineering to the sciences and mathematics. As time went on, our work with the youngsters began to acquire a character all its own; many questions were raised, some of which our colleagues in Hungary[2] *and the*

USSR had faced before us, some which represented new questions and problems set for us by the rapidly changing world.

Thanks to the enthusiastic support of our teacher-students and the assistance of many university colleagues who learned about our experiment, the number of able high school participants grew from year to year until it stabilized at about seventy (70). This limit was imposed primarily by the available material resources. The summer of 1962 was quite typical. Our group of participants consisted of 73 boys and girls who came from 18 states of the USA, representing 71 secondary schools (high schools), and was distributed in a very wide age range from 14 to 17 years of age.

We have found that the selection of mathematical content as well as the mode of treatment determine the quality of involvement of our young charges.

Whenever the selection of participants relies on some measure of their potential, judged through the quality of the initiative, the intensity of their interest and the degree of their perseverance, one is bound to bring together a group with a wide spectrum of interests. At the outset, therefore, one must confront the question of what purpose should be served by a mathematics program for a collection of young individuals who have in common only eagerness, curiosity, an unbounded (and hitherto undirected) supply of vitality and, possibly, an ultimate destiny in science.

One may decide upon the selection of some mathematical skills generally considered useful in the sciences. One may choose some applications falling within the range of the student's experience and jointly treat the mathematical tools and the ideas in this selected field of application. One may, on the other hand, take advantage of the fact that *with a proper choice of material one can remain within mathematics and yet exhibit a whole gamut of problems which confronts every scientist.*

Even though we did not intend to neglect any of the above alternatives entirely, at the outset (the first summer of our multi-level program) we subordinated everything to the aim of providing our young charges with the opportunity of acquiring experience in scientific thinking while remaining within the field of mathematics. We have never regretted this choice of strategy.

Since 1957, we have used number theory as the basic vehicle for the development of the student's capacity for observation, invention, the use of language, and all those traits of character which constitute intellectual discipline.

We have chosen number theory because of its wealth of accessible, yet fundamental and deep mathematical ideas and for its wealth of challenging but tractable problems. Our choice has turned out to be practical. In our treatment of number theory, we make use of the fact that this subject can lead to the study of modern algebra and hard analysis. We have also found that an appropriate treatment of number theory lays the groundwork for the study of combinatorics and discrete mathematics generally, a range of mathematical ideas which is becoming increasingly important in science and technology.

Although the problem-solving experience which is acquired by the student who prepares himself for competitive examinations is valuable, it tends to give a mistaken idea that examination scores provide a measure of intellectual development.[3] *We have felt that the education of future scientists should also encourage the kind of involvement which develops the student's capacity to observe keenly, to ask astute questions and to* *recognize significant problems. This last factor is important in scientific education for two reasons. First, the progress of every science depends upon the capacity of its practitioners to ask penetrating questions and to identify important problems. Second, we believe that personal discovery is a vital part of the learning process for every individual eager to gain deep insight into his subject.*

Thus, as we work at number theory, our aim is to develop attitudes as well as skills.[4] *From the very beginning the participant is given an opportunity to develop his powers of observation, to experiment and to discover significant relations between the objects of his experimentation. He learns to use counter-examples to destroy untenable conjectures, and as his experience grows, he begins to provide the security of a proof for the surviving conjectures. As is natural in all fields of human activity, word labels follow the recognition of phenomena; the incentive for the precise and concise use of language comes from the desire to share one's experience with others.*

Every opportunity is taken to point out and make use of relationships between arithmetic, on the one hand, and algebra, geometry and analysis on the other. A strong effort is made to provide the participant with a very large number of problems, many of which call for considerable ingenuity. Extension of some basic results in the arithmetic in Z to polynomials and to some quadratic fields is undertaken as an exercise of insights and methods put together under the title "techniques of generalization."[5] *Whenever possible, constructive methods are used, and the student is encouraged to master available algorithms.*

I will not repeat the account given in Mathematical Scientist of topics discussed in our multilevel program prior to 1977. I will try to update this *account by mentioning some of the interesting topics introduced since then and intended for advanced participants.*

Those of our program participants who respond well to what we do together in the first summer are invited to return to the program in the following summers. In working with them we try to build on the experience which they acquired in their first summer. Topics are chosen on the basis of their importance in mathematics and science and on the availability of accomplished senior people who are vivid expositors and who are interested in young people.

In the summers of 1980, 1981, 1982 and 1984, a basic course in Combinatorics (Monday through Friday), taught by Professors Tom Dowling and Neil Robertson of OSU, was open to our more experienced participants. In 1983, this course was taught by Professor Daniel Hughes of London, England. In 1985, 1986 and 1987, this course was taught by Professor Dijen Ray-Chaudhuri of OSU.

1980: A triple track experimental course in Analysis consisting of a course in Real Calculus (Track I, MWF), taught by Professor Bogdan Baishanski; a course in P-adic Calculus (Track II, MWF), taught by Professor Kurt Mahler of the Institute of Advanced Studies, A.N.U., Canberra, Australia; and a problem seminar (T, Th, S) where problems from both Track I and Track II were discussed and where the students could observe the similarities and the essential differences between these two important versions of calculus. This seminar was directed by Professor Ranko Bojanic. Track II dealt with much new elementary material from the second edition of Professor Mahler's Cambridge Tract.

1982: A course in Combinatorial Geometry done for our program by Professors Dan Burghelea (OSU) and Guido Mislin (Zurich).

<u>1983</u>: A course in Projective Planes taught by Professor Daniel Hughes, and two mini courses, each of about three weeks duration, were designed for the advanced program participants:

A. Some Fundamental Ideas of Topology by Professor Dan Burghelea (OSU). This sequence of lectures introduced the students to the spirit and the methods of "TOPOLOGY". The Euler-Poincare characteristic of polyhedra, an important numerical measure of topological complexity, formed the central theme.

B. Motions in Hyperbolic Plane Geometry and the Classification of Surfaces by Professor Guido Mislin (ETH Zurich). The topics treated were an introduction to non-Euclidean geometry, the Poincare model for the Hyperbolic plane, Mobius transformation groups of hyperbolic translations, and classification of compact surfaces.

<u>1985</u>: An introduction to Representation Theory of Finite Groups designed for our advanced participants was taught by Professor Dan Burghelea. Our chemistry colleague, Professor Bruce Bursten, described how a chemist makes use of the group character theory in the study of the behavior of substances with a known molecular structure. This provided a vivid example of interaction between mathematics and science.

<u>1986</u>: A course in Probability taught by Professor Antoine Brunel (University of Paris) for our advanced participants. He began by discussing discrete probability models and their combinatorial properties, then considered continuous probability models by assuming the existence of densities and using intuitive analogies with volumes to replace the technicalities of calculus. This made it possible the discussion of such

sophisticated applications as random walks, central limit theorems, Brownian motion, and fractal sets.

We have felt for some time that our most experienced participants, particularly those who come to us for a second or third summer in succession and whose interests definitely point in the direction of mathematics and science, should have an opportunity to gain some appreciation of the subtler concerns of the experimentalist. They should also acquire some feeling, within the limitations of their experience, for the preoccupations of the theoreticians of science. We have been fortunate that our scientific colleagues have been willing to help us in this difficult task. Here we have much to learn.

We have introduced a lecture series intended to provide an appreciation of current movement in mathematics and in the changing relationship between science and mathematics.

In 1984, Professor Felix Browder introduced this series by a lecture entitled, "Mathematics and Science."

In 1986 Professor Victor Klee gave three lectures on "Unsolved Problems in Discrete Mathematics."

In 1987, Professor Charles Fefferman gave three lectures on "The Mathematics of Quantum Theory." These lectures were based on his paper "N-body Problem in Quantum Mechanics."

All of these lectures are intended primarily for our advanced participants. Each summer we provide an opportunity for them to discuss the mathematical ideas which are basic for the understanding of the lectures. The lectures are scheduled for the last week of our summer program sessions.

In the case of programs put together in collaboration with our colleagues abroad, the range of the subject matter studied depended upon the duration of each program.

In our India (pilot) program of three weeks duration (All India Summer Institute in Mathematics, Bangalore, 1973) directed by Professor V. Krishnamurthy of Birla Institute of Technology and Science in Pilani, we were able to bring within the reach of our students an impressive selection of topics in number theory thanks to the assistance of our colleagues, Professor Krishnamurthy himself and Dr. T. Sonndarajan of Madurai University.

In our two-week Australian Summer (January) Program in Canberra, directed since 1969 by Professor Larry Blakers of the University of Western Australia, we have used number theory as a systematic introduction to mathematical thinking in each (Australian) summer since 1975. Each summer we were learning how to induce an ever deeper involvement of our young charges.

A mathematics program similar to our U.S. program was held at the Mathematics Institute of the University of Heidelberg at the end of summer (mid-August to mid-September) in 1978 and 1979. The program was directed by Professor Peter Roquette.

The duration of our U.S.A. Summer Program is eight weeks. In it we provide sufficiently varied mathematical activity to make it worthwhile inviting some of its participants to return for the second or even third summer.

Eight weeks gives us sufficient time to allow for reasonable scope and depth in our treatment of number theory. Naturally each summer there are variations of emphasis in the treatment of different sections due to a *variation of student response to different topics, or to the needs of colleagues discussing some related ideas.*

The program was invited to the University of Chicago in 1975 where Professor Felix Browder served as its director until the summer of 1977.

It took four summers to establish our Columbus program at the University of Chicago. The Chicago program was doing very well from the very beginning and acquired good momentum by the summer of 1978. At the request of the OSU Department of Mathematics, we have revived our summer mathematics program at The Ohio State University as of the summer of 1979. We have continued our program at The Ohio State University each summer since that time.

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[1] *Ross, A. E., Mathematical Scientist, 3, 1-7 (1978).*

[2] *Ross, Arnold E. (1973). Fostering scientific talent. Chapter V of S Science and Technology Policies (Ballinger Publishing Co., Cambridge, Mass.).*

[3] *Ross, Arnold E. (1987). Problem-solving - Its environment and its significance.*

[4] *Engel, A., Mathematisohe Schulerwettbewerbe, Jehrbuch Uberblicke Mathematik, 1979. Wissensohaftsverlag Bibliographisohes Institut, Mannhein.*

[5] *Ross, A. E., Proceedings of the Fourth International Congress on Mathematical Education, 4,698-699 (1980).*