UNIVERSITY OF NOTRE DAME DU LAC

NOTRE DAME, INDIANA

"TO TEACH HOW TO TEACH HOW TO DO"

The Notre Dame Mathematics Teacher Training Program

Summer, 1963

Duration: June 18 - August 3, 1963

The Notre Dame Mathematics Teacher Training Program

1947 - 1963

Introduction

It has been generally recognized for a long time that there exists a pressing need for a happy union of two extremes in the training of mathematics teachers - the one extreme emphasizing the methods of teaching to the exclusion of the consideration of the mathematical content, and the other extreme allowing an early preoccupation with the highly technical detail of some specialized mathematical subject with the consequent neglect of the broader mathematical culture so essential for the effective teaching of mathematics.

The present summer teacher training program at Notre Dame is an attempt to provide the needed happy combination.

In response to many questions by our friends we would like to make it clear that it is not our intention to deny the relevance of or to belittle the importance of the extramathematical problems of teaching.

We do feel that in teaching mature people it is a sound practice to make these people aware of the problems and the difficulties involved in communicating to them the ideas and the skills of the subject taught. These are truly considerations of pedagogy and we believe that in making these an integral part of our teaching of the subject matter we are less likely to be overwhelmed by trivialities and are more likely to produce a deeper and a more lasting impression upon the student-teacher. It is thus that we propose to introduce the student-teacher to the problems inherent in the communication of ideas.

We believe that the process by which an individual acquires mastery of a subject is essentially the same as the process by which the understanding of that subject has been achieved through the efforts of many people over a long period of time. We are convinced that the act of personal discovery through observation and experimentation is a vital ingredient of learning and

should be an inevitable concomitant of every effective scheme for the communication of ideas and of intellectual skills.

I. The Student Body

Our program began in 1947 with a small group of high-school and junior-college teachers. The student body has grown steadily since then. In the summer of 1956, there were 84 graduate students in the mathematics department (all of whom were practicing high-school or college teachers) working toward an M. S. degree in mathematics and 22 students in the related science programs who took some of the courses in the mathematics teacher training program.

A new impetus to the development of our program has been provided by the National Science Foundation Grants for High -School Teachers for the summers of 1957, 1958, 1959, 1960, 1961 and 1962. In the summer of 1961, the number of students participating in the Teacher Training Program totaled 247. A total of 172 participants were supported wholly or in part by the National Science Foundation. In the summer of 1962 the number of participants in our Teacher Training Program totaled 273 (most of whom were practicing high-school teachers). A total of 201 participants were supported wholly or in part by the National Science Foundation.

With the help of the National Science Foundation, we once again hope to be able to make it possible for a significant number of high-school teachers to participate in our program. We expect the enrollment in the summer of 1963 to be about 300.

Many of the teacher-students in our program come in order to pursue systematically for five summers either the program outlined in detail in the section on curriculum or a combination of this program with similar programs of study in other fields, such a combination leading to an M. S. in Mathematics with a possible minor in some related field. Six teachers completed their work for an M. S. in 1961. Twenty-three participants in the National Science Foundation Academic Year Institute completed their work for an M. S. in the summer of 1961. In the summer of 1962, twenty teachers completed their M. S. work. Twenty-two participants in the National Science

Foundation Academic Year Institute also earned their M. S. in the summer of 1962.

We do have a number of students who come only for a summer or two to listen to lectures in some topic or topics of immediate importance to them in their teaching.

Now and then in our summer teaching we come across students of exceptional mathematical ability who would not have come into contact with advanced mathematical ideas outside of a summer program such as ours. We encourage such students to come during the regular academic year and to pursue a much more intensive course of studies.

At times visitors with more advanced training come to observe and to comment upon our work in teacher training. We welcome such visitors and hope that their number will increase in the future. We have been much encouraged by the sympathetic interest in our work shown by such visitors in the past.

II. Duration

Our Mathematics Teacher Training Program takes place during a seven week summer-school session at Notre Dame. In 1963, our summer session will start on June 18, and will end on August 3. Registration will take place on June 19.

III. Objectives

It is the aim of the Mathematics Teacher Training Program to bring the high school and college teachers into close contact with the best traditions of mathematics and of the teaching of mathematics. We believe that if teachers succeed in assimilating such traditions through an effective program of thoughtful mathematical instruction, then they will perpetuate these same traditions in their own teaching.

Our mathematics courses in the Teacher Training Program are not courses in the methods of teaching mathematics. Our courses contain a

carefully prepared introduction to those ideas of higher mathematics which are of particular value to the teacher of mathematics.

We feel that our Teacher Training Program must fulfill two conditions: First, through an enlightened realistic plan of training it should develop in our students greater competence in their chosen field and through that, greater confidence in themselves and a greater interest in their work. Second, in order to be truly successful, our curriculum should represent the best mathematical thought of our time. Toward this end, we are making a determined effort to interest outstanding mathematicians in our work and in coming here during the summer to teach what they would consider a good teachers' course in their chosen fields.

IV. (1) The Curriculum

Students without advanced mathematical training are advised to follow closely the plan outlined below. For students with some experience in mathematics a suitable modification is worked out upon consultation.

First Summer

Mathematics 123s - Elementary Number Theory

The algebra of classes. Basic properties of the ring of integers, divisibility, Euclid's Algorithm, prime numbers, factorization into primes, congruences. Diophantine equations. Congruences with one unknown.

In addition to the usual basic concepts of set theory the content is approximately that of Chapers I - VII of Uspensky and Heaslet's "Elementary Number Theory".

Mathematics 228s - Higher Algebra I

Professor Walter Ledermann
Summer, 1962

Basic mathematical concepts (mappings, relations, etc.).
Groups, rings, integral domains, fields, polynomial rings and their fundamental properties.

The level of precision is approximately that of Birkhoff and MacLane "A Survey of Modern Algebra".

In teaching our "First Summer" students, we set as our principal objectives the development of the precise use of language, the correct use of the processes of mathematical reasoning, and, last but not least, some appreciation of the art of mathematical discovery and of problem solving. A strong effort is made through the use of numerous examples and exercises to give the student an adequate background for the understanding of abstract mathematical concepts and the student is encouraged to participate in the problem seminars related to the courses.

Second Summer (Two out of the three courses listed below)

Mathematics 139s - Numerical Analysis

Concept of real numbers. Decimal expansion. Cantor expansion. Continued fraction expansion. Test for irrationality. Periodic continued fractions. Approximations of real numbers by rationals. Approximation of algebraic numbers. Liouville's numbers. Approximation of functions by polynomials. Tchebysheff's polynomials.

Mathematics 229s - Higher Algebra II

Vector spaces. Matrices and linear transformations. Determinants, volumes, and systems of linear equations.

The level of precision is approximately that of Schreier and Sperner's "Einfurung in die Analytische Geometrie und Algebra".

Mathematics 281s - Geometry I

A critical re-examination of the fundamental concepts of geometry.

Although in the case of the "Second Summer" student it is possible to teach more difficult mathematical ideas by building upon the foundations laid during the student's first summer at Notre Dame, still much more emphasis must be placed upon the consolidation of the gains in the mastery of the fundamentals achieved during the first summer.

Third Summer (The choice of courses is made on the basis of the student's mathematical background. In addition to the courses listed under the second summer, we recommend the courses described below).

Mathematics 140s - Analysis I

Professor W. W. Rogosinski
Summer, 1960

This course concentrates on a few basic concepts in analysis, with some emphasis on working experience with sequences and series. The topics discussed are: I. The development of the system of real numbers (treated as a complete ordered field) starting with Peano's postulates. II. Sequences (Cauchy convergence criterion, limits superior and inferior, Bolzano-Weierstrass theorem). III. Infinite series (convergence tests, absolute convergence, rearrangements, Cauchy product).

Mathematics 282s - Geometry II

Continuation of Geometry I. The level of treatment approximates that of Veblen and Young, Vol. I.

Fourth and Fifth Summers (Electives)

Mathematics 201s - Classical Mechanics

This course provides a logical development of analytical dynamics. In particular it deals with kinematics, Newton's laws, dynamics of a particle, planetary motion, dynamics and statistics of rigid bodies, gyrostatic problems, Lagrange's equations, theory of small oscil-

lations, Hamilton's equations and Hamilton's principle.

Mathematics 230s - Elementary Set Theory

An introductory course in set theory. Fundamental set-theoretical notions. Schroeder-Bernstein Theorem. Cardinal and ordinal numbers. Choice axiom and the continuum hypothesis. The application of set theory to the various mathematical problems.

Professor B. Sobocinski, University of Notre Dame
Summer, 1962

Mathematics 234s - Combinatorial Problems.

Starting from the combinatorial notion of operation tables, in particular Latin squares, the properties of binary operations of finite systems were studied. The structure of finite groups and semi-groups, rings and fields was discussed, and properties of these structures not commonly discussed in the basic modern algebra courses were considered. Problem sets developed a very wide variety of isomorphic representations of finite structures.

Professor Thoralf Skolem
Summer, 1962

The above discussion was followed by a brief sketch of algebraic number fields, and concepts of integers, primes, units, norms in algebraic number fields and in the algebra of Quaternions.

Dirichlet's principle (Schubfachschluss) was introduced and a series of applications made of it to obtain upper bounds for the least non-trivial solution in integers of systems of linear equations or congruences. This was again used to express positive integers as sums of squares and to solve in integers the homogeneous ternary quadratic equation. Further, Dirichlet's principle was used in the study of Pell's equation, in particular the proof of a theorem which makes the solution of some exponential equations possible.

At the last the triple systems of Steiner and similar combinatorial subjects were treated.

Mathematics 247s - Geometrical Theory of Numbers

Topics in the applications of geometry to the theory of numbers selected to fit the background of the student group.

Professor Kurt Mahler
Summer, 1962

Mathematics 250s - Analysis II

Fundamental ideas and theorems on continuity, derivatives, Riemann integrals and series of functions.

Mathematics 251s - Analysis III

Limits and Series. A detailed study of infinite series and of the basic topics in the theory of functions.

Professor W. W. Rogosinski
Summer, 1962

Mathematics 253s - Irrational Numbers

It is the objective of this course to make the student feel at home with the basic ideas and methods of analysis by bringing him into contact with individual (and famous) examples of irrational numbers and by giving him an opportunity of working with a variety of special methods of approximation of irrationals by rational numbers (such as g-adic expansions, Cantor expansions, continued fraction expansions, and infinite products). As a by-product the student comes into contact with a rich sample of good mathematical ideas in number theory and algebra as well as analysis.

Professor H. D. Kloosterman
Summer, 1957
Professor Kurt Mahler
Summer, 1959

Mathematics 291s - Mathematical Logic

The basic concepts of propositional and predicate calculus, that is, truth-tables, axiomatization consistency and independence of axiom sets, completeness, validity and sat-

Father Ivo Thomas, Black Friars College, Oxford.
Summer, 1962

isfiability are studied. An appreciation of formal proof is stressed. Some applications are given.

Mathematics 335s - Elementary Analytic Number Theory

Professor S. Chowla
Summer, 1958

Mathematics 385s - Topics in Analysis

IV (2) Lecture Series

Lecture Series (a). Curriculum

In the summer of 1963, we plan to continue the series of lectures and seminars devoted to the discussion of experimental high-school curricula and experimental undergraduate college curricula.

Each summer we invite people deeply concerned with the problems of the revision of high-school curricula and with the related problem of Teacher Training, to visit us and to explain their ideas. Of these one may mention Professors Max Beberman, Norman T. Hamilton, Joseph Landin and Herbert Vaughan of the "Illinois Group"; Professor Robert Davis of the "Madison Project"; Dean Francis Keppel of the Harvard Graduate School; Professor David Page of the Illinois Elementary Project; and the Honorable John Brademas, Member of Congress, Member of Congressional Education Committee.

We were able to provide for many participants an opportunity for an informal discussion with these distinguished educators.

The National Science Foundation sponsored inspection tours by visitors from abroad intended to encourage the international exchange of ideas and methods in secondary education and in teacher training. We at Notre Dame were very happy to play host to Professor B. A. Amira of the University of Jerusalem; Professor Jorge Carbonell Borbonet of

Montevideo, Uruguay; Professor Richard Guy of the University of Malaya; Professor Barros Pereira of Rio de Janeiro, Brazil and Professor R. P. Bambah of Punjab University, Chandigarh, India.

Lecture Series (b). Teaching over TV. Visual and other aids to teaching

We were able to begin our Lecture Series (b) in the Summer of 1958. Our experiment consisted of a series of nine lectures over closed circuit TV to the group of over 250 participating mathematics teachers and other interested students and members of the University Staff. The program was viewed over nine television receivers. Members of the TV audience came to the studio on a rotation plan so that almost everyone was able to get a close look at the mechanics of the broadcasts.

There was round table discussion on the lessons of this experiment and an evaluation of the written comments is planned for the near future.

We have continued our TV experiments and a detailed report on what we have done is being prepared.

Lecture Series (c) Creative Mathematics

We plan to continue our program of lectures and problem seminars under the title "Creative Mathematics". Each series of one to ten lectures is to be self-contained and is to discuss some research problems accessible by elementary methods. The seminar will be devoted to the discussion of problems on the intermediate and advanced levels. The aim of this program is to emphasize the living - the dynamic - nature of mathematics. Visiting lecturers in this program have been Professors A. A. Albert, Alfred Brauer, Hans Jonas, Walter Ledermann, Wilhem Magnus, E. J. McShane, Marston Morse, Paul Rosenbloom, W. Warwick Sawyer, John Todd, and Herbert Vaughan.

IV. (3) The Staff

Our 1962 senior staff consisted of Professors Ranko Bojanic, Norman Haaser, Milko Jeglic, George Kolettis, A. E. Ross (Director of the Program) and B. Sobocinski; Visiting Professors Abraham Goetz (Wroclaw, Poland), Walter Ledermann (Manchester, England), Horst Leptin (Hamburg, Germany), Kurt Mahler (Manchester, England), W. W. Rogosinski (Aarhus, Denmark), Thoralf Skolem (Oslo, Norway), and the Very Reverend Ivo Thomas, O. P. (Oxford, England); and Sisters Barbara Ann Foos, S. S. J. (Nazareth College, Rochester, New York), and Mary Robert Von Wolff, O. P. (Saint Mary's College, New Orleans, Louisiana).

We hope that most of the permanent members of the Notre Dame staff who taught in the summer of 1962 will again teach in the summer of 1963. In particular we expect that Professor Walter Ledermann will teach a second course in algebra, Professor Rogosinski a course in Analysis, Professor Thoralf Skolem a course in Combinatorial Problems, Professor Sobocinski a course in Elementary Set Theory, and Father Ivo Thomas a course in Mathematical Logic.

IV. (4) Our Visitors

In the early days of our existence we were fortunate in having a number of very accomplished mathematicians come and give one and sometimes several lectures to our students in the Teacher Training Program. Of these one may mention Professor Richard Brauer, Professor Paul Erdős, Professor Solomon Lefschetz and Professor Harry Vandiver.

With particular reverence and appreciation one recalls the summer term when the late Professor Max Dehn taught a course in Projective Geometry in our program.

V. Academic Credit

Each course meets five times a week, Monday through Friday, for a

full clock hour and carries a credit of three semester hours. Most courses also have an associated problem seminar meeting three hours a week.

Stipend holders in the National Science Foundation Summer Institute do not pay tuition and are entitled to credit for each successfully completed course, provided that their interest in such credit is indicated at registration time.

For students with appropriate qualifications not holding a National Science Foundation stipend academic credit is available up to six (6) semester credit hours at the cost of $165.00 in tuition, and up to $20.00 in other fees and costs (Lab. fees, textbooks).

The degree of an M. S. in mathematics can be earned upon the successful completion of ten suitable courses (30 semester hours of credit) with the grade average of 4 or over (on the scale of 1 up to 6) and the passing of a comprehensive examination.

VI. Counseling

In 1957, as an extension of our counseling program we were able to arrange fourteen (14) problem seminars for students in our basic courses such as Number Theory, Higher Algebra I and Geometry I. We increased the number of problem seminars in the summer of 1958 to nineteen (19) and in 1959 the number was increased to twenty-six (26). In 1960 and 1961 the number of problem seminars was increased to thirty-four (34). In 1962 we had thirty-five (35) problem seminars.

A very great emphasis is laid upon the counseling of students.

VII. Facilities

A. A first rate mathematics and science library. The Mathematics Library is housed in the Nieuwland Science Building in a well equipped reading room with a professionally trained library staff

for assistance to students and faculty in library reference work. The Mathematics Library has a good working collection both for undergraduate instruction and for research.

Special attention has been given to the collection of mathematical research journals from many countries and in many languages. Not only are most of the currently published journals received but back files of all the important journals are maintained.

In 1960, 1961 and 1962 we were able to place at the disposal of our students a large library reading room containing many mathematics books of particular interest to teachers.

B. A modern bookstore which sponsors large textbook exhibits.

C. Modern classrooms.

D. The campus itself is practically a park with two lakes, shaded areas, walks, etc. Recreation is available for men on the golf course, swimming in the lake, tennis courts, etc. Free concerts, etc.

E. Housing is ample on campus for both men and women students.

F. Housing for married couples can be arranged in South Bend.

G. The University cafeteria is open to the public from 7 30 a. m. to 7:00 p. m. each day.

H. Facilities for informal social gatherings such as faculty-student teas.

The foregoing is a bare outline of the activities of our Teacher Training Program. It is difficult to describe in words the spirit which is created through the working together of an eager, interested and devoted student body and a dedicated faculty.

Arnold E. Ross

Arnold E. Ross, Head
Department of Mathematics and
Director of the Program.