

Poster Session Titles and Abstracts (Alphabetical Order)

Sarah Bray (Tufts)

Ergodic geometry for nonstrictly convex Hilbert geometries

In his work on convex, open, bounded Ω in projective space with compact quotients by discrete groups of projective transformations, Benoist proves that in many ways the strictly convex Ω generalize hyperbolic space when endowed with the Hilbert geometry. Dynamically, he and Crampon have independently shown that the geodesic flow on the unit tangent bundle of the quotient manifold exhibits classical uniformly hyperbolic behavior (it is an Anosov flow) and hence the measure of maximal entropy is unique.

For a nonstrictly convex Hilbert geometry, new techniques are needed. Benoist proved under mild hypotheses that for nonstrictly convex Hilbert geometries in dimension three, compact quotients have embedded flats as boundary tori or Klein-bottles to hyperbolic components. We explore how techniques from rank one manifolds and nonuniformly hyperbolic dynamics need to be and can be adapted for these examples, with emphasis on the power of geometry for proving hyperbolic dynamical phenomena. Main results include positive entropy, existence and uniqueness of conformal measures at infinity, divergence of the group, and ergodicity of Bowen-Margulis measure for the geodesic flow.

Marta Canadell (ICERM, Brown)

Computation of normally hyperbolic invariant manifolds

In this poster we explain a method for the computation of normally hyperbolic invariant manifolds (NHIM) in discrete dynamical systems. The method is based on finding a parameterization for the manifold by formulating a functional equation. We solve the invariance equation using a Newton-like method taking advantage of the dynamics and the geometry of the invariant manifold and its invariant bundles. Particularly, we present two different methods to compute normally hyperbolic invariant tori (NHIT). The first method is a KAM-like theorem in a-posteriori format for the existence of quasi-periodic invariant tori in smooth families of real-analytic dynamical systems, which provides us an efficient algorithm for computing NHIT, by adjusting parameters of the family. The second method allows us to compute a NHIT and its internal dynamics, which is a-priori unknown. We implement both methods to continue invariant tori with respect to parameters, and to explore different mechanisms of breakdown. This is a joint work with Alex Haro.

Dong Chen

Positive metric entropy arises near a flat metric on 5-dimensional torus

In 2014, D. Burago and S. Ivanov constructed a Finsler metric on S^n ($n > 3$) that is C^∞ close to the standard metric but with positive metric entropy. In this poster we show that similar case happens on torus. Namely, for arbitrary n , we can perturb the flat metric on T^5 in the class of C^n reversible Finsler metric so that the resulting metric has positive metric entropy.

Xue Gong (Ohio University)

Heteroclinic binding networks—a model in sequential memory

Temporal order memories are critical for everyday animal and human functioning. Some evidence has shown that the binding or association of various features of an event together and the maintaining of multimodality events in sequential order is the key component of any sequential memories. We studied the mathematical aspect of this using a high-dimensional model in the generalized form of the Lotka-Volterra equations. In the phase space of the model there exists a multi-dimensional binding heteroclinic network consisting of a heteroclinic chain of heteroclinic cycles. We proved the robustness of the binding sequential dynamics, i.e., the feasibility phenomenon for coupled heteroclinic networks: for each collection of successive heteroclinic trajectories inside the unified networks, there is an open set of initial points such that the trajectory going through each of them follows the prescribed collection staying in a small neighborhood of it. We show also that the symbolic complexity function of the system restricted to this neighborhood is a polynomial. Therefore, the symbolic system is not chaotic.

Charles Jaffé (West Virginia University)

Relative Equilibria in the Semiclassical Coulomb problem

I discuss the nature of the Semiclassical Coulomb problem and then turn my attention the relative equilibria in these systems. I review what is known concerning relative equilibria in atoms and molecules. I then turn my attention to the role that relative equilibria play in chemical reactions.

John Lesieutre (UIC)

Which Three-Dimensional Algebraic Varieties Admit Positive Entropy Automorphisms?

There are many examples of smooth, algebraic surfaces admitting (biholomorphic) automorphisms of positive entropy. However, there are few known examples of such automorphisms on higher-dimensional varieties. I will present some results which partly explain this scarcity.

Bingbing Liang (Buffalo)

Sofic mean topological dimension of algebraic actions

For any discrete group Γ , there is a one-to-one correspondence between $\mathbb{Z}\Gamma$ -modules M and the topological dynamical systems $\Gamma \curvearrowright \widehat{M}$ by continuous automorphisms (so-called *algebraic actions*). The sofic mean topological dimension is a numerical invariant in dynamics and is related to entropy. The von Neumann-Lück rank is a L^2 -invariant and is related to L^2 -Betti number.

Assume Γ is a sofic group, M is a finitely-presented $\mathbb{Z}\Gamma$ -module such that \widehat{M} is an absolute neighborhood retract (ANR), we show that the sofic mean topological dimension of \widehat{M} equals to the von Neumann-Lück rank of M . Although the condition of being an ANR does not hold in general, we show it has some connection to the expansiveness of dynamical systems.

Olga Lukina (UIC)

The discriminant invariant of Cantor group actions.

Abstract: In this paper, we consider minimal equicontinuous actions of non-abelian groups on Cantor sets. Such an action Φ may be classified as *regular*, *weakly regular* or *irregular* by the properties of its automorphism group, which reflects the degree of non-homogeneity of the action. We introduce an invariant, called the *discriminant function* $\mathcal{D}(\Phi)$, which assigns to each point $x \in X$ a profinite group \mathcal{D}_x . The cardinality of \mathcal{D}_x does not depend on the point and is related to the degree of non-homogeneity. We characterize the actions for which the discriminant group is finite, and give new examples of minimal actions for which \mathcal{D}_x is not trivial. Joint work with Jessica Dyer and Steve Hurder.

Lien-Yung Kao (Notre Dame)

Entropy, critical exponent and immersed surfaces in hyperbolic 3-manifolds

We consider a π_1 -injective immersion $f : \Sigma \rightarrow M$ from a compact surface Σ to a hyperbolic 3-manifold M . Let Γ denote the copy of $\pi_1\Sigma$ in $\text{Isom}(\mathbb{H})^3$ induced by the immersion f , and we endow Σ with the induced metric. Using the thermodynamic formalism, when Σ is negatively curved and Γ is convex cocompact, we prove an inequality relating the topological entropy $h(\Sigma)$ of the geodesic flow on T^1M and the critical exponent δ_Γ of Γ : $C_1(\Sigma, M) \cdot \delta_\Gamma \leq h(\Sigma) \leq C_2(\Sigma, M) \cdot \delta_\Gamma$, where $C_1(\Sigma, M)$ and $C_2(\Sigma, M)$ are two geometric constants. Herein, we investigate geometry meanings of these two constants in detail. Furthermore, we study the rigidity phenomenon arising from this inequality. Lastly, we apply our results to immersed minimal surfaces in hyperbolic 3-manifolds, and these discussions lead us to a similar study as A. Sanders' work on the moduli space of Σ introduced by C. Taubes.

Mehrzad Monzavi (University of Texas at Arlington)

On Fixed Point Property of Nilpotent and Solvable Lie Group Actions on Nonpositively Curved Compact Manifolds

We will prove the following theorems. The first theorem posits the existence of a fixed point for the actions of nilpotent lie groups on nonpositively curved compact manifolds. The second theorem states that actions of solvable Lie groups on nonpositively curved compact manifolds have either a fixed point or a 2-periodic point.

Cara Mullen (UIC)

p-adic Hubbard Trees

In complex dynamics, Hubbard trees offer a combinatorial description of the dynamics of post-critically finite (PCF) polynomials. What are the analogous objects in a non-Archimedean setting: what is a p -adic Hubbard tree? We explore this question by studying the critical orbit trees associated to quadratic maps $f_c(z) = z^2 + c$, with $c \in \mathbb{Z}_p$ (for $p > 2$).

Rob Niemeyer (University of Maine)

Nontrivial paths and periodic orbits of the T-fractal billiard.

In this poster presentation, we: 1) determine some periodic orbits and nontrivial paths of the T -fractal billiard table, 2) construct a forward singular orbit that behaves like particular periodic orbits, and 3) present future directions of research.

Anca Radulescu (SUNY New Paltz)

Real and complex behavior for networks of coupled logistic maps

Many natural systems are organized as networks, in which the nodes interact in a time-dependent fashion. The object of our study is to relate connectivity to the temporal behavior of a network in which the nodes are real or complex logistic maps, coupled according to a connectivity scheme that obeys certain constraints, but also incorporates random aspects. We discuss the possibility of extending Fatou-Julia theory to such ensembles of maps coupled as nodes in a network.

We investigate in particular the relationship between the system architecture and dynamics. We illustrate how the system's behavior (measured via topological properties of the Julia set) changes when perturbing the underlying adjacency graph. We differentiate between the effects on dynamics of the following operations which directly modulate network connectivity: (1) increasing/decreasing edge weights, (2) increasing/decreasing edge density, (3) altering edge configuration by adding, deleting or moving edges.

Yuki Takahasi (UC-Irvine)

Products of two Cantor sets and application to the Labyrinth model

We consider products of two Cantor sets, and obtain the optimal estimates in terms of thickness that guarantee that their product is an interval. This problem is motivated by the fact that the spectrum of the Labyrinth model, a two dimensional quasicrystal model, is given by the product of two Cantor sets. We also discuss the connection between this question and "intersection of two Cantor sets" problem, which was considered in several papers before.