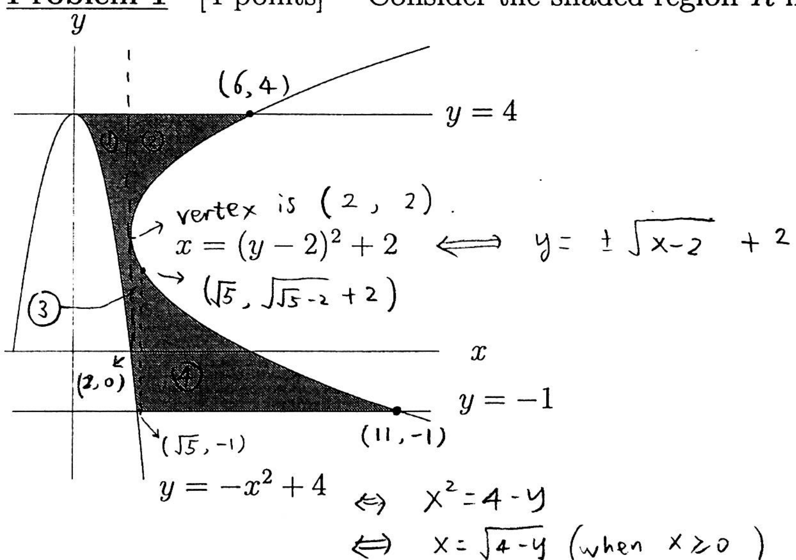
Math 1172 - Autumn 2015 Name: ______________________

Quiz 1 - Take Home Recitation Instructor: ______________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [4 points] Consider the shaded region R in the image below.



a) Set up an integral that represents the area of the region R using the y-axis.

$$\text{area}(R) = \int_{-1}^{4} (y-2)^2+2 - \sqrt{4-y}\; dy$$

b) Set up a sum of integrals that represents the area of the region R using the x-axis.

Making vertical lines $x=2$ and $x=\sqrt{5}$ to split R into parts ①, ②, ③ and ④

$$\text{area}(R) = \text{area}(①) + \text{area}(②) + \text{area}(③) + \text{area}(④)$$

$$= \int_0^2 4-(-x^2+4)\,dx + \int_2^6 4-(\sqrt{x-2}+2)\,dx$$

$$+ \int_2^{\sqrt{5}} (-\sqrt{x-2}+2)-(-x^2+4)\,dx + \int_{\sqrt{5}}^{11} (-\sqrt{x-2}+2)-(-1)\,dx$$

c) Compute and compare the values for the area of region R from parts a and b.

See the next page.

a) $\text{area}(R) = \int_{-1}^{4} (y-2)^2 + 2 - \sqrt{4-y}\, dy$

$\int (y-2)^2 + 2\, dy = \frac{(y-2)^3}{3} + 2y + C$, $\int \sqrt{4-y}\, dy \underset{du=-dy}{\overset{u=4-y}{=\!=}} \int -\sqrt{u}\, du = -\frac{2}{3} u$

$\int \sqrt{4-y}\, dy \underset{du=-dy}{\overset{u=4-y}{=\!=}} \int -\sqrt{u}\, du = -\frac{2}{3} u^{\frac{3}{2}} + C = -\frac{2}{3}(4-y)^{\frac{3}{2}} + C$

So $\text{area}(R) = \frac{(y-2)^3}{3} + 2y + \frac{2}{3}(4-y)^{\frac{3}{2}} \Big|_{-1}^{4}$

$= \frac{8}{3} + 8 - \left(\frac{-27}{3} - 2 + \frac{2}{3} \cdot 5^{\frac{3}{2}}\right) = \frac{65}{3} - \frac{2 \cdot 5^{\frac{3}{2}}}{3}$

b). $\text{area}(①) = \int_0^2 x^2 dx = \frac{x^3}{3}\Big|_0^2 = \frac{8}{3}$.

Using

$\int \sqrt{x-2}\, dx \underset{du=dx}{\overset{u=x-2}{=\!=}} \int \sqrt{u}\, du = \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{3}(x-2)^{\frac{3}{2}} + C$,

we have

$\text{area}(②) = \int_2^6 2 - \sqrt{x-2}\, dx = 2x - \frac{2}{3}(x-2)^{\frac{3}{2}} \Big|_2^6$

$= 12 - \frac{2}{3} \cdot 4^{\frac{3}{2}} - (2 \cdot 2 - 0) = 8 - \frac{2 \cdot 4^{\frac{3}{2}}}{3} = 8 - \frac{2 \cdot 8}{3}$

$\text{area}(③) = \int_2^{\sqrt{5}} -\sqrt{x-2} + x^2 - 2\, dx = -\frac{2}{3}(x-2)^{\frac{3}{2}} + \frac{x^3}{3} - 2x \Big|_2^{\sqrt{5}}$

$= -\frac{2}{3}(\sqrt{5}-2)^{\frac{3}{2}} + \frac{5^{\frac{3}{2}}}{3} - 2\sqrt{5} - \left(0 + \frac{8}{3} - 4\right)$

$= \frac{-2}{3}(\sqrt{5}-2)^{\frac{3}{2}} + \frac{5^{\frac{3}{2}}}{3} - 2\sqrt{5} - \frac{8}{3} + 4$

$\text{area}(④) = \int_{\sqrt{5}}^{11} -\sqrt{x-2} + 3\, dx = -\frac{2}{3}(x-2)^{\frac{3}{2}} + 3x \Big|_{\sqrt{5}}^{11}$

$= -\frac{2 \cdot 27}{3} + 33 - \left(-\frac{2}{3}(\sqrt{5}-2)^{\frac{3}{2}} + 3\sqrt{5}\right)$

So $\text{area}(R) = \frac{8}{3} + 8 - \frac{2 \cdot 8}{3} - \frac{2}{3}(\sqrt{5}-2)^{\frac{3}{2}} + \frac{5^{\frac{3}{2}}}{3} - 2\sqrt{5} - \frac{8}{3} + 4$

$- \frac{2 \cdot 27}{3} + 33 + \frac{2}{3}(\sqrt{5}-2)^{\frac{3}{2}} - 3\sqrt{5}$

$= 12 + 33 - 18 - \frac{16}{3} + \frac{5^{\frac{3}{2}}}{3} - 5\sqrt{5}$ $\quad (5\sqrt{5} = 5^{\frac{3}{2}})$

$= \frac{65}{3} - \frac{2 \cdot 5^{\frac{3}{2}}}{3}$. So, we get the same answer from (a) and (b).