Journal of Financial Economics 103 (2012) 411–427

Contents lists available at SciVerse ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

Endogenous technological progress and the cross-section of stock returns ☆

Xiaoji Lin *

Department of Finance, London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK

ARTICLE INFO

Article history:
Received 19 January 2009
Received in revised form
14 October 2010
Accepted 15 November 2010
Available online 5 September 2011

JEL classification:
E23
E44
G12

Keywords:
Technological progress
R&D investment
Physical investment
Stock return

ABSTRACT

I study the cross-sectional variation of stock returns and technological progress using a dynamic equilibrium model with production. Technological progress is endogenously driven by research and development (R&D) investment and is composed of two parts. One part is devoted to product innovation; the other, to increasing the productivity of physical investment. The latter is embodied in new tangible capital. The model breaks the symmetry assumed in standard models between tangible and intangible capital, in which the accumulation processes of tangible and intangible capital stock do not affect each other. Qualitatively and, in many cases, quantitatively, the model explains well-documented empirical regularities.

1. Introduction

Using a dynamic equilibrium model, this paper investigates intangible capital, tangible capital, and the cross-section of stock returns. The paper focuses primarily on intangible capital in the form of firms' research and development (R&D) efforts.[1] The central insight of the paper is that physical capital—embodied technological progress is essential to simultaneously explain the well-documented puzzling facts regarding R&D investment and physical investment: High R&D-intensive firms earn higher average stock returns than low R&D-intensive firms (e.g., Chan, Lakonishok, and Sougiannis, 2001; Li, forthcoming), and high physical investment—intensive firms earn lower average stock returns than low physical

☆ This paper is based on Chapter 1 of my doctoral dissertation at the University of Minnesota. I am grateful to my committee members Frederico Belo, John Boyd, Murray Frank, Tim Kehoe, and especially Robert Goldstein (chair) and Lu Zhang for their valuable advice and continual encouragement. I thank Antonio Bernardo, Laurent Fresard, John Kareken, Sam Kortum, Ellen McGrattan, Pedram Nezafat, Monika Piazzesi, Lukas Schmid, Chun Xia, and Suning Zhang for their comments. I thank the seminar participants at Arizona State University, Indiana University, London School of Economics and Political Science, New York University, University of Michigan, University of Minnesota, University of Toronto, University of Washington at Seattle, and the 2007 Financial Management Association Doctoral Students Consortium and the Western Finance Association 2008 meetings. I thank Ryan Israelsen for sharing the quality-adjusted investment goods price series. I acknowledge the support of the Carlson School Dissertation Fellowship. All errors are my own.

* Tel.: +44 20 7852 3717; fax: +44 20 7955 7420.
E-mail address: x.lin6@lse.ac.uk

0304-405X/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.jfineco.2011.08.013

[1] Tangible capital consists primarily of equipment, machines, and plants, and it is usually labeled as physical capital. Throughout the paper, I use the terms tangible and physical and the terms intangible and R&D interchangeably.

investment—intensive firms (e.g., Titman, Wei, and Xie, 2004; Xing, 2008).[2] Moreover, this paper directly links technological innovation to the differences between value and growth firms, thereby providing a fresh explanation for the value premium that differs from the existing literature.

The positive covariation between R&D investment and expected stock returns is puzzling for the neoclassical q-theory of investment. As Cochrane (1991) and Liu, Whited, and Zhang (2009) show, under constant returns to scale technology, stock returns equal investment returns. Because investment negatively forecasts expected investment returns, it must also be negatively correlated with expected stock returns. However, this prediction is inconsistent with R&D's positive forecasting of expected stock returns.

Standard models cannot simultaneously explain the different covariations between R&D investment, physical investment, and expected stock returns. For example, Hansen, Heaton, and Li (2004) and McGrattan and Prescott (2005) treat tangible capital and intangible capital symmetrically. More specifically, in their respective models, the accumulation processes of tangible and intangible capital stock do not affect each other. However, these models predict that R&D investment and physical investment forecast future stock returns in the same direction, which is counterfactual.

The model in this paper is based on three underlying assumptions. The first is that technological progress is endogenously driven by R&D investment. This assumption is familiar from Romer (1990), who argues that technological progress largely arises from firms' R&D investment. In the model, I assume that technological progress is a result of firms' explicit R&D decisions and is represented by intangible capital. Here, intangible capital primarily refers to successful innovations in advances in manufacturing technologies and processes, new designs and formulas that generate new products, and so on.

The second assumption is that part of firms' technological progress is devoted to new products. This assumption comes from the R&D literature. Cohen and Klepper (1996) and Lin and Saggi (2002) find that a large proportion of firms' R&D expenditures is used in innovations to generate new products. In pharmaceuticals, software companies, for example, more than half of total R&D expenditures is dedicated to new product innovations. Typically, product innovation increases firms' cash flows through the introduction of new product features that increase the price buyers are willing to pay for firms' products or that allow firms to reach new buyers.[3] In the model, product innovations combined with physical capital produce products.

The third assumption is the key assumption in the model: The other part of technological progress is innovation devoted to increasing the productivity of physical investment in producing new physical capital. Hence, in the model, the advances of new physical capital embody current technological progress. This assumption is crucial to simultaneously generate a positive covariation between R&D investment and future stock returns and a negative covariation between physical investment and future stock returns. The assumption of embodiment captures the fact that successful innovations increase the productivity of equipment and machines and reduce production costs (Levin and Reiss, 1988; Cohen and Klepper, 1996). In petroleum refining, biochemical industry, for example, more than two-thirds of total R&D expenditures is dedicated to innovations in reducing production costs. Likewise, a number of other industries, including petrochemicals, food and beverage manufacturing, and semiconductor plants, invest R&D in manufacturing technology for designing, analyzing, and controlling manufacturing through timely measurements (during processing) of critical quality and performance attributes of raw and in-process materials and processes, with the goal of ensuring final product quality.

The main economic implications of the model are as follows. First, firms' expected returns on physical investment are increasing in R&D investment but decreasing in physical investment. Intuitively, expected physical investment return is the ratio of the expected marginal benefit of physical investment to the marginal cost of physical investment. All else being equal, on the one hand, R&D investment increases the expected marginal benefit of physical investment; on the other hand, R&D investment (physical investment) decreases (increases) the marginal cost of physical investment. These two effects reinforce each other and imply that R&D investment (physical investment) increases (decreases) the expected returns on physical investment.

The second economic implication is that high R&D-intensive firms earn higher expected stock returns than low R&D-intensive firms, whereas high physical investment—intensive firms earn lower expected stock returns than low physical investment—intensive firms. Intuitively, in the model, the stock price is the sum of the market value of physical capital and R&D capital, and the stock return is the weighted average of physical investment return and R&D investment return. Because physical capital embodies current technological progress (R&D capital) and its share in output production dominates that of R&D capital, the market value of physical capital is higher than the market value of R&D capital. This relation implies that the weight on the physical investment return is greater than the weight on the R&D investment return.[4] Therefore, firms' stock returns covary with R&D investment and physical investment in the same way that physical investment returns do. The implication is that stock returns are increasing in R&D investment but decreasing in physical investment.

[2] Hsu (2009) finds that, at an aggregate level, the cumulative R&D growth rate positively forecasts future stock market returns. Cochrane (1991) and Lamont (2000) find that aggregate physical investment also negatively forecasts future stock market returns.

[3] Firms with new product can usually raise prices through some degree of transient monopoly power.

[4] In the model, the weight on the physical (R&D) investment return is the ratio of the market value of physical (R&D) capital to the stock price.

The third economic implication is that value firms earn higher expected stock returns than do growth firms. Intuitively, with a high book-to-market ratio, value firms have low physical investment, which implies that they must earn high expected physical investment returns. Growth firms have low expected physical investment returns because they have high physical investment with a low book-to-market ratio. Hence, value firms earn high expected stock returns, whereas growth firms earn low expected stock returns because the weight on the physical investment return is larger than the weight on the R&D investment return. More specifically, in the model, the productivity of the existing physical capital of value firms is lower than that of growth firms, because value firms invest less in R&D. In recessions, value firms are burdened with excessive physical capital and do not have as much technological progress in upgrading the efficiency of the existing physical capital as do growth firms, so they are more risky given that the market price of risk is high in bad times. The value premium in my model hinges on the interactions between technological progress and physical investment. This mechanism differs from that in Zhang (2005), who works through physical capital adjustment costs in generating the value premium. Given that most of the studies on book-to-market ratio and stock returns focus only on physical investment, this paper sheds light on the relation between technological progress and the value premium.

My work is closely related to a growing strand of literature that studies asset pricing in production economies. See, for example, Cochrane (1991, 1996), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Cooper (2006), and Belo (2010). In contrast to the existing literature, which focuses on physical investment and expected stock returns, this paper explores the relation between firms' technological progress, physical investment, and the cross-sectional variation of returns.

In a paper similar to mine, Li (forthcoming) constructs a dynamic real options model in which R&D investment and stock returns change in predictable ways when R&D firms are financially constrained. The key distinction between my model and Li's is that Li's real options model features exogenous cash flows, systematic risk, and financing constraints, whereas my model employs a neoclassical framework in which technological progress is endogenously determined. Hence, in my model the key economic fundamental variables, i.e., R&D investment, physical investment, and stock returns, are determined endogenously in competitive equilibrium. My model can, therefore, shed light on the fundamental determinants of technological progress and the covariations between R&D investment, physical investment, and future stock returns without resorting to financing frictions.

This study also differs from much of the literature addressing technological progress in macroeconomics and growth analyses. I do not examine the link between R&D investment and growth, as addressed by the endogenous growth models (e.g., Romer, 1990; Aghion and Howitt, 1992), in which physical capital goods are produced by R&D and raw capital. Although I do use a similar modeling device, the difference is that endogenous growth models employ a monopolistic, competitive equilibrium (Dixit and Stiglitz, 1977) to represent the physical capital goods sector and to capture the idea that output exhibits increasing returns to scale in R&D. New ideas represent a different variety of new physical capital goods. My model differs because I focus on the economic mechanism that R&D capital increases the productivity of physical investment in producing new physical capital. Notably, Albuquerque and Wang (2008) use investment-specific technological change to examine the asset pricing and welfare implications of imperfect investor protection at an aggregate level. This paper differs from Albuquerque and Wang's in that it focuses on the implications of firms' technological change on asset prices and returns.

This paper also relates to the literature that integrates financial frictions to corporate investment, q-theory, and asset pricing, e.g., Hennessy and Whited (2007), Livdan, Sapriza, and Zhang (2009), Bolton, Chen and Wang (2011), DeMarzo, Fishman, He, and Wang (forthcoming), etc. The key difference is that this paper focuses on the asset pricing implications of firms' investment in technological innovations; financial frictions are not explicitly studied.

2. The model

The equilibrium model I present here is constructed with production, aggregate uncertainty, and firm-specific uncertainty. The economy comprises a continuum of competitive firms that produce a homogeneous product, taking the product price as given.

2.1. Technology

Production requires two inputs, physical capital, k, and R&D capital, n, and is subject to both an aggregate shock, x, and an idiosyncratic shock, z. The aggregate productivity shock has a stationary and monotone Markov transition function, denoted by $Q_x(x_{t+1}|x_t)$, as

$$x_{t+1} = \overline{x}(1-\rho_x) + \rho_x x_t + \sigma_x \varepsilon^x_{t+1}, \tag{1}$$

where ε^x_{t+1} is an independently and identically (IID) standard normal shock.

The idiosyncratic productivity shocks, denoted by $z_{j,t}$, are uncorrelated across firms, indexed by j, and have a common stationary and monotone Markov transition function, denoted by $Q_z(z_{j,t+1}|z_{j,t})$, as

$$z_{j,t+1} = \rho_z z_{j,t} + \sigma_z \varepsilon^z_{j,t+1}, \tag{2}$$

where $\varepsilon^z_{j,t+1}$ is an IID standard normal shock and $\varepsilon^z_{i,t+1}$ and $\varepsilon^z_{j,t+1}$ are uncorrelated for any pair (i, j) with $i \neq j$. Moreover, ε^x_{t+1} is independent of $\varepsilon^z_{j,t+1}$ for all j.

In the model, the aggregate productivity shock is the driving force behind economic fluctuations and systematic risk, and the idiosyncratic productivity shock is the driving force behind the cross-sectional heterogeneity of firms.

The production function is constant returns to scale in physical capital and intangible capital:

$$y_{j,t} = e^{x_t + z_{j,t}}(k_{j,t})^{\alpha}(\theta n_{j,t})^{1-\alpha}, \tag{3}$$

where $y_{j,t}$ is output; $k_{j,t}$ is physical capital stock including structures, equipment, and machines; $n_{j,t}$ is R&D (intangible) capital stock for firm j at period t including innovations in designs and formulas, new technologies in manufacturing, etc.; and θ is the proportion of R&D capital devoted to producing final products with $0<\theta\leq 1$.

The percentage of total R&D dedicated to product innovation varies across industries. In pharmaceuticals, fabricated and metal products, software programming, for example, more than three-quarters of R&D is used for product innovation. Meanwhile in industries such as petroleum refining, biochemicals, semiconductors, and electrical equipment, product innovation is less than 30%. As shown in Cohen and Klepper (1996), a large part of such variation is due to differences in exogenous industry-level conditions that systematically differentiate the returns to one sort of innovative activity from another.[5]

In the rest of the paper, I drop the firm index j when no confusion results.

2.2. Intangible capital and tangible capital production

This subsection presents the accumulation dynamics of intangible and tangible capital.

2.2.1. Endogenous technological progress

As is standard in the literature, intangible capital production follows the standard capital accumulation process given by

$$n_{t+1} = (1-\delta_n)n_t + o_t, \tag{4}$$

where o_t is R&D investment. Standard models also commonly assume that physical capital follows a symmetric process, $k_{t+1} = (1-\delta_k)k_t + i_t$, where i_t is physical investment.

However, specifying physical capital and intangible capital symmetrically produces a model that predicts that both R&D investment and physical investment forecast expected stock returns in the same direction, which is counterfactual. I, therefore, abandon the symmetry of standard models and specify the following accumulation process for physical capital:

$$k_{t+1} = (1-\delta_k)k_t + \Phi[i_t,(1-\theta)n_{t+1}], \tag{5}$$

where

$$\Phi[i_t,(1-\theta)n_{t+1}] \equiv A\{a(i_t)^{\rho} + (1-a)[(1-\theta)n_{t+1}]^{\rho}\}^{1/\rho} \tag{6}$$

is a constant elasticity of substitution (CES) technology for physical capital production.[6]

Here, $\{a,\rho,A\}$ are constants with the constraints $0<a\leq 1$, $\rho<1$, $\rho\neq 0$, and $A>0$. Note that $(1-\theta)n_{t+1}$ is the proportion of intangible capital dedicated to producing new physical capital. The CES function $\Phi[i_t,(1-\theta)n_{t+1}]$ in Eq. (5) generalizes the standard accumulation process as a special case when $A=a=\rho=1$. It satisfies $\Phi_1[i_t,(1-\theta)n_{t+1}]>0$, $\Phi_2[i_t,(1-\theta)n_{t+1}]>0$, $\Phi_{11}[i_t,(1-\theta)n_{t+1}]<0$, $\Phi_{12}[i_t,(1-\theta)n_{t+1}]>0$, and $\Phi_{22}[i_t,(1-\theta)n_{t+1}]<0$, where numerical subscripts denote partial derivatives. That is, the total product of physical capital increases in the level of physical investment and intangible capital. Moreover, the marginal product of physical investment decreases in physical investment but increases in intangible capital, and the marginal product of R&D capital decreases in R&D capital but increases in physical investment.[7] Economically, with the restriction $\rho<1$, both Φ_{12} and Φ_{21} being positive implies that physical investment i_t and R&D capital n_{t+1} are substitutes. This property is important to generate the positive relation between R&D investment and stock returns. The elasticity of substitution between n_{t+1} and i_t is $1/(1-\rho)$.

The most important aspect of Eq. (5) is the inclusion of the intangible capital $(1-\theta)n_{t+1}$, which represents the current state of technological progress for producing new physical capital. A high realization of $(1-\theta)n_{t+1}$ increases the productivity of physical investment and directly upgrades the efficiency of physical capital from the current vintage to the next. The increases in $(1-\theta)n_{t+1}$ formalize the notion of embodied technological progress.

2.2.2. Motivation

The motivations for Eq. (5) come from the macroeconomic literature on investment-specific technological change.[8] Theoretically, as shown in Greenwood, Hercowitz, and Krusell (1997, 2000), technological progress, such as faster and more efficient means of telecommunications and transportation, new and more powerful computers, robotization of assembly lines, the advances of

[5] With θ being constant, the model effectively rules out the cross-industry variation of returns driven by the different composition of firms' R&D efforts, which could create an identification issue for cross-sectional heterogeneity. To address this issue, one can endogenously model the composition of firms' R&D activities by allowing firms to optimally choose the share of product innovation. Doing so will allow for an analysis of the various effects of different R&D uses on stock returns. However, this analysis is beyond the scope of this paper and better left for future research. I thank the referee for pointing out this issue.

[6] The CES production function in Eq. (6) contains several well-known production functions as special cases, depending on the value of parameter ρ. For instance, when $\rho=1$, $\Phi[i_t,(1-\theta)n_{t+1}]$ is a linear production function; when $\rho\to 0$, $\Phi[i_t,(1-\theta)n_{t+1}]$ is the Cobb-Douglas technology; when $\rho\to-\infty$, $\Phi[i_t,(1-\theta)n_{t+1}]$ reduces to the Leontif technology.

A theoretical justification for the functional form of $\Phi[i_t,(1-\theta)n_{t+1}]$ can be found in Huffman (2007), who assumes an adjustment cost function for the production of new physical capital, which is dependent on both physical investment and intangible capital. Huffman employs a functional form similar to CES and characterizes it as $\Psi(i_t,n_t)=B[(1-\theta)(i_t)^{\rho/(1-\theta)}+\theta(n_t)^{\rho}]^{1/\rho}$. This function implies that the cost of producing new physical capital depends on the amount of intangible capital that has been previously undertaken. It is assumed that $\theta\leq 0$, so that the cost function is increasing in i_t and decreasing in n_t. That is, the larger the stock of research knowledge, the cheaper it is to produce a specific amount of new physical capital.

The cost function in the equation can be transformed into a production function (see Mas-Colell, Whinston, and Green, 1995, Subsections 5.C and 5.D, pp. 135–147) similar to the CES function in Eq. (6).

[7] $\Phi_{21}[i_t,(1-\theta)n_{t+1}]=\Phi_{12}[i_t,(1-\theta)n_{t+1}]$.

[8] A different label for capital-embodied technological change is investment-specific technological change. See Greenwood, Hercowitz, and Krusell (1997) for interpretations.

manufacturing technologies, and so on, has made production of new physical capital more efficient and less expensive. More specifically, Greenwood, Hercowitz, and Krusell assume that the physical capital accumulation process follows $k_{t+1}=(1-\delta_k)k_t+\phi_t i_t$, where ϕ_t is an exogenous technological progress different from the aggregate productivity shock. The technological progress ϕ_t determines the productivity of physical investment. In particular, it makes the new physical capital production more efficient by reducing the marginal cost of physical investment, which equals $1/\phi_t$ in equilibrium. Fisher (2006) estimates ϕ_t using the real equipment price and finds that ϕ_t is important to account for economic growth in both the short and long run in addition to the aggregate productivity shock. Huffman (2007) assumes that embodied technological progress is driven by R&D investment and reduces the adjustment costs of physical capital. In his model, economic growth takes place directly through aggregate R&D spending.

Eq. (5) can be rewritten as

$$k_{t+1}=(1-\delta_k)k_t+i_t\Phi_1[i_t,(1-\theta)n_{t+1}]+(1-\theta)n_{t+1}\Phi_2[i_t,(1-\theta)n_{t+1}], \tag{7}$$

where the equality follows from the fact that $\Phi[i_t,(1-\theta)n_{t+1}]$ is constant returns to scale in $(i_t,(1-\theta)n_{t+1})$. So the role of intangible capital $(1-\theta)n_{t+1}$ in Eq. (5) can be interpreted in two ways. First, $1/\Phi_1[i_t,(1-\theta)n_{t+1}]$ can be considered as representing the cost of producing a new unit of physical capital in terms of final output using physical investment only. This cost decreases in n_{t+1}. In other words, one can imagine that in each period, a new vintage of physical capital is produced by physical investment. The productivity of a new unit of physical investment is given by $\Phi_1[i_t,(1-\theta)n_{t+1}]$, which is increasing in $(1-\theta)n_{t+1}$. Second, $\Phi_2[i_t,(1-\theta)n_{t+1}]$ can be considered as representing the productivity of a new unit of intangible capital $(1-\theta)n_{t+1}$ in producing new physical capital k_{t+1}. This productivity $\Phi_2[i_t,(1-\theta)n_{t+1}$ increases in i_t. In sum, technological progress makes new physical capital either less expensive or better than old physical capital, allowing for increased output.

I endogenize ϕ_t in Greenwood, Hercowitz, and Krusell (1997, 2000) by assuming technological progress occurs at the level of firms and is a result of firms' R&D decisions in Eq. (5). To get a transparent relationship between R&D capital in the model and the investment-specific technological change in Greenwood et al., I let $\rho\to 0$ so that $\Phi[i_t,(1-\theta)n_{t+1}]$ reduces to the standard Cobb-Douglas technology, which effectively makes R&D capital and physical investment perfectly substitutable. With further simplifications, the physical capital accumulation dynamics in the model can be rewritten as

$$k_{t+1}=(1-\delta_k)k_t+n_{t+1}i_t.^9 \tag{8}$$

It follows that there is a one-to-one mapping between R&D capital n_{t+1} and the investment-specific technological change ϕ_t in Greenwood, Hercowitz, and Krusell.[10] Intuitively, the efficiency of physical capital can be upgraded through time, and R&D decisions involve determining how much more efficient to make the next vintage of capital. The economic implication is that R&D capital drives the investment-specific technological change.

To test this prediction, I examine the relationship between the level of investment-specific technological change and the aggregate real R&D capital as the firm-level R&D stock is not readily available. Real R&D capital is the net R&D stock from the Bureau of Economic Analysis (BEA), deflated by the consumer price index. Investment-specific technological change is the level of technology in producing equipment capital, which is calculated as the ratio of the quality-adjusted price of equipment and the constant-quality price index for consumption. The Hodrick-Prescott (HP)-filtered real R&D capital and the level of investment-specific technology covary positively with a correlation coefficient of 0.41 (0.01), where the number in parentheses is the p-value. This finding confirms the model's prediction: R&D capital and the investment-specific technological change co-move through time. More important, this finding suggests that the economic driving force of the investment-specific technological change can be firms' R&D investment decisions. The firms' R&D capital makes equipment evermore efficient than that of the previous generation. Therefore, Eq. (5) provides a direct microfoundation for the investment-specific technological change in the macroliterature and offers rich interactions between the current technological progress n_{t+1} and physical investment i_t.

2.2.3. Obsolescence of physical capital

In practice, the advance of technology makes the existing tangible capital obsolete. For example, computers make typewriters obsolete. In the model, technological progress makes physical capital obsolete by increasing its economic depreciation.

Eq. (7) can be rewritten as

$$\frac{k_{t+1}}{\Phi_{1,t}}=\frac{(1-\delta_k)}{\Phi_{1,t}/\Phi_{1,t-1}}\frac{k_t}{\Phi_{1,t-1}}+i_t+(1-\theta)n_{t+1}\frac{\Phi_{2,t}}{\Phi_{1,t}} \tag{9}$$

and

$$\tilde{k}_{t+1}=(1-\tilde{\delta}_k)\tilde{k}_t+i_t+(1-\theta)n_{t+1}\frac{\Phi_{2,t}}{\Phi_{1,t}}, \tag{10}$$

where

$$\tilde{k}_{t+1}=\frac{k_{t+1}}{\Phi_{1,t}} \tag{11}$$

and

$$(1-\tilde{\delta}_k)=\frac{(1-\delta_k)}{\Phi_{1,t}/\Phi_{1,t-1}}. \tag{12}$$

Eq. (9) follows from dividing both sides of Eq. (7) by $\Phi_{1,t}$. The numerical subscript denotes the partial derivative of $\Phi(\cdot)$ with respect to its argument.

[9] The simplifications follow by setting $\theta=0$, $a=1-a$ (i.e ., $a=1/2$), and then normalizing $A=a^{-1/\rho}$ and letting $\rho\to 0$ in the CES technology in Eq. (6) be such that it reduces to standard Cobb-Douglas technology, i.e., $\Phi[i_t,(1-\theta)n_{t+1}]=n_{t+1}i_t$. Combining Eq. (5) and this equation, I get $k_{t+1}=(1-\delta_k)k_t+n_{t+1}i_t$ as in Eq. (8).

[10] Without the above simplifications, one can still derive a positive relation between R&D capital and investment-specific technological change when comparing the benchmark model with Greenwood, Hercowitz, and Krusell. With $\rho\to 0$, the relation is transparent.

Physical capital stock $\tilde{k}_{t+1}$ is now measured (at market value) in terms of physical investment. With this new measurement, a unit of new physical capital can be considered as being $\Phi_{1,t}/\Phi_{1,t-1}$ more productive than a unit of old physical capital. Therefore, when new physical capital is produced, the market value of old physical capital is reduced by $1/(\Phi_{1,t}/\Phi_{1,t-1})$. Hence, in Eq. (10), $\tilde{\delta}_k$ represents the economic rate of depreciation for physical capital, whereas δ_k is the accounting rate of depreciation. Intuitively, $\Phi_{1,t}$ is the productivity of one unit of physical investment in producing new vintage physical capital, which is increasing in the current state of technological progress, n_{t+1}. The higher the technological progress n_{t+1}, the bigger the productivity $\Phi_{1,t}$, and the larger the economic depreciation rate $\tilde{\delta}_k$ ($\tilde{\delta}_k > \delta_k$), i.e., the higher the degree of obsolescence of existing tangible capital induced by technological progress.

2.3. Stochastic discount factor

Following Zhang (2005), I directly specify the pricing kernel without explicitly modeling the consumer's problem. The pricing kernel is given by

$$\log M_{t,t+1} = \log \beta + \gamma_t (x_t - x_{t+1}) \tag{13}$$

and

$$\gamma_t = \gamma_0 + \gamma_1 (x_t - \overline{x}), \tag{14}$$

where $M_{t,t+1}$ denotes the stochastic discount factor from time t to $t+1$. The parameters $\{\beta, \gamma_0, \gamma_1\}$ are constants satisfying $1 > \beta > 0$, $\gamma_0 > 0$, and $\gamma_1 < 0$.

Eq. (13) can be motivated as a reduced-form representation of the intertemporal marginal rate of substitution for a fictitious representative consumer. In particular, consistent with Zhang (2005), I assume in Eq. (14) that γ_t is time varying and decreases in the demeaned aggregate productivity shock $x_t - \overline{x}$ to capture the countercyclical price of risk with $\gamma_1 < 0$.[11]

2.4. Dynamic value maximization

I assume that firms own their capital and are financed purely by equity. As such, once investment has been made, the residual is distributed as a dividend, d_t, i.e.,

$$d_t = y_t - i_t - o_t.^{12} \tag{15}$$

Let $v(k_t, n_t, x_t, z_t)$ denote the cum-dividend market value of the firm. I state the firm's dynamic value maximization problem as

$$v(k_t, n_t, x_t, z_t) = \max_{k_{t+1+j}, n_{t+1+j}, i_{t+j}, o_{t+j}} \mathbb{E}_t \sum_{j=0}^{\infty} M_{t,t+j} d_{t+j}$$

$$\text{s.t. Eqs. (4) and (5) with } k_t, n_t \text{ given.} \tag{16}$$

[11] The precise economic mechanism driving the countercyclical price of risk is, e.g., time-varying risk aversion as in Campbell and Cochrane (1999).

[12] A negative dividend is considered as equity issuance.

2.5. Solutions

In this subsection, I present the solutions of the model.

2.5.1. First-order conditions

Let q_t^n and q_t^k be the present value multiplier associated with Eqs. (4) and (5). The first-order conditions with respect to i_t, o_t, k_{t+1}, and n_{t+1} are, respectively,

$$q_t^k = 1/\Phi_1[i_t, (1-\theta)n_{t+1}], \tag{17}$$

$$q_t^n = 1, \tag{18}$$

$$q_t^k = \mathbb{E}_t\{M_{t,t+1}[\alpha y_{t+1}/k_{t+1} + (1-\delta_k) q_{t+1}^k]\}, \tag{19}$$

and

$$1-(1-\theta) q_t^k \Phi_2[i_t, (1-\theta)n_{t+1}] = \mathbb{E}_t\{M_{t,t+1}[(1-\alpha) y_{t+1}/n_{t+1} + 1 - \delta_n]\}, \tag{20}$$

where $\Phi_1[i_t, (1-\theta)n_{t+1}]$ and $\Phi_2[i_t, (1-\theta)n_{t+1}]$ are the partial derivatives of $\Phi[i_t, (1-\theta)n_{t+1}]$ with respect to its first and second argument.

Eqs. (17) and (18) are the optimality conditions for physical investment and R&D investment that equate the marginal costs of investing in physical capital and intangible capital, $(1/\Phi_1[i_t, (1-\theta)n_{t+1}]$ and 1), with their marginal benefits, (q_t^k and q_t^n). Here, (q_t^k, q_t^n) are known as the marginal q of physical investment and R&D investment, respectively. The direct marginal cost of R&D investment at the optimum is one, but with an indirect benefit of $(1-\theta) q_t^k \Phi_2[i_t, (1-\theta)n_{t+1}]$, the effective marginal cost (effective marginal q) of R&D investment, denoted as $\tilde{q}_t^n$ hereafter, is $1-(1-\theta) q_t^k \Phi_2[i_t, (1-\theta)n_{t+1}]$. Economically, the term of $(1-\theta) q_t^k \Phi_2[i_t, (1-\theta)n_{t+1}]$ is the value of marginal product of R&D capital in producing new tangible capital. All else equal, physical investment i_t increases the value of marginal product of R&D capital $(1-\theta) q_t^k \Phi_2[i_t, (1-\theta)n_{t+1}]$ and, hence, reduces the effective marginal cost of R&D investment $\tilde{q}_t^n$. That is, the higher the physical investment i_t, the lower the effective cost of R&D investment in producing intangible capital. This is the channel through which physical investment (capital) can affect the production of intangible capital.

Eqs. (19) and (20) are the Euler equations that describe the optimality conditions for physical capital and R&D capital. Intuitively, Eqs. (19) and (20) state that to generate one additional unit of physical capital and intangible capital at the beginning of next period, (k_{t+1}, n_{t+1}), a firm must pay the price of physical capital and intangible capital (equal to the marginal q of physical investment and the effective marginal q of R&D investment at the optimum), $(1/\Phi_1[i_t, (1-\theta)n_{t+1}], 1-(1-\theta) q_t^k \Phi_2[i_t, (1-\theta)n_{t+1}])$. The next-period marginal benefit of this additional unit of physical capital and intangible capital includes the marginal product of capital, $(\alpha y_{t+1}/k_{t+1}, (1-\alpha) y_{t+1}/n_{t+1})$, and the marginal continuation value of physical capital and intangible capital net of depreciation, $((1-\delta_k) q_{t+1}^k, 1-\delta_n)$, respectively.

2.5.2. Investment returns and stock return

To derive the asset-pricing implications from the model, I define one-period returns for physical investment and R&D investment based on Eqs. (19) and (20):

$$r_{t+1}^k \equiv \frac{\alpha y_{t+1}/k_{t+1}+(1-\delta_k)q_{t+1}^k}{q_t^k} \tag{21}$$

and

$$r_{t+1}^n \equiv \frac{(1-\alpha)y_{t+1}/n_{t+1}+1-\delta_n}{\tilde{q}_t^n}. \tag{22}$$

Intuitively, the investment (both physical and R&D) return from time t to time $t+1$ is the ratio of the marginal benefit of investment at time $t+1$ divided by the marginal cost of investment at time t. I also define the one-period stock return as

$$r_{t+1}^s \equiv \frac{p_{t+1}^s+d_{t+1}}{p_t^s}, \tag{23}$$

where p_t^s is the ex-dividend stock price.

Proposition 1 establishes a link between the firm's market value, the firm's expected stock return, and the firm's physical investment and R&D investment in closed form.

Proposition 1. The ex dividend stock price, p_t^s, equals the sum of the market values of physical capital and intangible capital. The stock return is a weighted average of the physical investment and R&D investment returns:

$$p_t^s = q_t^k k_{t+1} + \tilde{q}_t^n n_{t+1}, \tag{24}$$

$$r_{t+1}^s = \frac{q_t^k k_{t+1}}{p_t^s} r_{t+1}^k + \frac{\tilde{q}_t^n n_{t+1}}{p_t^s} r_{t+1}^n. \tag{25}$$

Proof. See Appendix A.

Intuitively, the market value of the equity of a firm is made up of the market values of two economic fundamentals, physical capital and intangible capital. Accordingly, the return on equity consists of the returns on these two economic fundamentals. The market value decomposition is simply an extension of the result of Hayashi (1982) to a multifactor inputs setting. Galeotti and Schiantarelli (1991) and Merz and Yashiv (2007) provide similar results.

2.5.3. Marginal q for intangible capital

In the model, the marginal q for intangible capital is one because there are no frictions for intangible capital accumulation. Proposition 1 can be used to test this prediction empirically. If Eq. (24) is estimated at the firm level, two issues arise.

1. Intangible capital n_{t+1} is not measurable at the firm level. Although the existing literature (e.g., Chan, Lakonishok, and Sougiannis, 2001) can be followed to measure R&D capital assuming straight-line depreciation, this approach generates measurement error in estimated R&D capital, which biases the results.
2. In the model, the effective marginal $\tilde{q}_t^n$ is not one. Instead, it is $\tilde{q}_t^n = 1-(1-\theta)q_t^k \Phi_2[i_t,(1-\theta)n_{t+1}] = 1-((1-a)(1-\theta)^\rho/a)(i_t/n_{t+1})^{1-\rho} < 1$. $\tilde{q}_t^n$ can be directly identified, but $q_t^n = 1$ cannot be identified in the data.

Despite these issues, estimating the model could be an interesting topic for future research. One can construct predicted stock prices and stock returns following Cochrane (1991) and Liu, Whited, and Zhang (2009). Doing so allows for an examination of whether the model with both intangible and tangible capital can help explain the asset-pricing anomalies.

2.5.4. Risk and expected stock return

In the model, risk and expected stock returns are determined endogenously along with firms' value maximization. Evaluating the value function in Eq. (16) at the optimum,

$$v(k_t,n_t,x_t,z_t) = d_t + \mathbb{E}_t[M_{t,t+1} v(k_{t+1},n_{t+1},x_{t+1},z_{t+1})] \tag{26}$$

and

$$\Rightarrow 1 = \mathbb{E}_t[M_{t,t+1} r_{t+1}^s], \tag{27}$$

where Eq. (26) is the Bellman equation for the value function and Eq. (27) follows from the standard formula for stock return $r_{t+1}^s = v(k_{t+1},n_{t+1},x_{t+1},z_{t+1})/[v(k_t,n_t,x_t,z_t)-d_t]$. If I define $p_t^s \equiv v(k_t,n_t,x_t,z_t)-d_t$ as the ex dividend market value of equity, r_{t+1}^s reduces to the usual definition in Eq. (23), $r_{t+1}^s \equiv (p_{t+1}^s + d_{t+1})/p_t^s$.

Now I rewrite Eq. (27) as the beta-pricing form, following Cochrane (2001, p. 19):

$$\mathbb{E}_t[r_{t+1}^s] = r_{ft} + \beta_t \zeta_{mt}, \tag{28}$$

where $r_{ft} \equiv 1/\mathbb{E}_t[M_{t,t+1}]$ is the real interest rate, β_t is the risk defined as

$$\beta_t \equiv \frac{-Cov_t[r_{t+1}^s, M_{t,t+1}]}{Var_t[M_{t,t+1}]}, \tag{29}$$

and ζ_{mt} is the price of risk defined as

$$\zeta_{mt} \equiv \frac{Var_t[M_{t,t+1}]}{E_t[M_{t,t+1}]}. \tag{30}$$

Eqs. (28) and (29) imply that risk and expected returns are endogenously determined along with optimal investment decisions. All the endogenous variables are functions of four state variables (the endogenous state variables, k_t and n_t, and two exogenous state variables, x_t and z_t), which can be solved numerically.

2.6. Intuition

I use the equivalence of stock returns and the weighted average of physical investment returns and R&D investment returns to provide the driving forces behind expected returns:

$$\underbrace{\mathbb{E}_t[r_{t+1}^s]}_{\text{Expected stock return}} = \overbrace{\frac{q_t^k k_{t+1}}{p_t^s}}^{\text{Weight on physical investment return}} \underbrace{\mathbb{E}_t[r_{t+1}^k]}_{\text{Expected physical investment return}} + \overbrace{\frac{\tilde{q}_t^n n_{t+1}}{p_t^s}}^{\text{Weight on R\&D investment return}} \underbrace{\mathbb{E}_t[r_{t+1}^n]}_{\text{Expected R\&D investment return}} \tag{31}$$

The justification for this approach is in Cochrane (1997) and Liu, Whited, and Zhang (2009), who show that average equity returns are well within the range of plausible parameters for average investment returns.[13]

Eq. (31) is useful for interpreting the empirical facts relating to R&D investment, physical investment, market-to-book ratio, and expected stock returns because it ties expected returns directly to firm characteristics. The equation implies that there are four variables affecting expected stock returns: the expected physical investment returns, the expected R&D investment returns, and their respective weights.

2.6.1. Physical investment returns, R&D investment returns, and stock returns

From the definition of the physical investment return in Eq. (21), the expected physical investment returns, $\mathbb{E}_t[r^k_{t+1}]$, is given by

$$\underbrace{\mathbb{E}_t[r^k_{t+1}]}_{\text{Expected physical investment return from period } t \text{ to } t+1} \equiv \frac{\overbrace{\mathbb{E}_t[\alpha y_{t+1}/k_{t+1}+(1-\delta_k)q^k_{t+1}]}^{\text{Expected marginal benefit of physical investment at period } t+1}}{\underbrace{q^k_t}_{\text{Marginal cost of physical investment at period } t}}. \tag{32}$$

The first implication is that $\mathbb{E}_t[r^k_{t+1}]$ is increasing in R&D investment but decreasing in physical investment. Two effects determine the physical investment returns: (1) the productivity effect, the marginal product of physical capital $\alpha y_{t+1}/k_{t+1}$ in the numerator; and (2) the investment effect, the marginal cost of physical investment q^k_t in the denominator.[14] In the numerator, on the one hand, R&D investment increases the expected marginal product of physical capital because R&D capital increases the productivity of physical capital all else equal.[15] On the other hand, physical investment decreases the expected marginal product of physical capital because the expected marginal product of physical capital is diminishing marginal returns in physical capital.[16] In the denominator, R&D (physical) investment decreases (increases) the marginal cost of physical investment.[17] The productivity effect and the investment effect reinforce each other and imply that R&D (physical) investment increases (decreases) the expected physical investment return.

In contrast, the expected R&D investment return, $\mathbb{E}_t[r^n_{t+1}]$, is decreasing in R&D investment but increasing in physical investment. From the definition of the R&D investment return in Eq. (22), the expected R&D investment return is given by

$$\underbrace{\mathbb{E}_t[r^n_{t+1}]}_{\text{Expected R\&D investment return from period } t \text{ to } t+1} \equiv \frac{\overbrace{\mathbb{E}_t[(1-\alpha)y_{t+1}/n_{t+1}+1-\delta_n]}^{\text{Expected marginal benefit of R\&D investment at period } t+1}}{\underbrace{\tilde{q}^n_t}_{\text{Effective marginal cost of R\&D investment at period } t}}. \tag{33}$$

All else equal, physical investment, which appears in the numerator, increases the marginal product of R&D capital $(1-\alpha)y_{t+1}/n_{t+1}$; and R&D (physical) investment, which appears in the denominator, increases (decreases) the effective marginal cost of R&D investment $\tilde{q}^n_t$.[18] These two effects imply that R&D (physical) investment decreases (increases) the expected R&D investment return.

Given that the expected physical investment return and the expected R&D investment return covary with R&D investment and physical investment oppositely, I need to investigate the weights on investment returns to determine whether the physical investment return or the R&D investment return dominates in the stock return. Because new physical capital embodies (part of) intangible capital and the share of physical capital in output production dominates the share of intangible capital (see details in subsection 3.4.2 for the simulation results), the market value of physical capital $q^k_t k_{t+1}$ is larger than the market value of intangible capital $\tilde{q}^n_t n_{t+1}$, which implies that the weight on the physical investment return $q^k_t k_{t+1}/p^s_t$ is greater than the weight on the R&D investment return $\tilde{q}^n_t n_{t+1}/p^s_t$. Therefore, the physical investment return multiplied by its weight, $(q^k_t k_{t+1}/p^s_t) r^k_{t+1}$, dominates the R&D investment return multiplied by its weight, $(\tilde{q}^n_t n_{t+1}/p^s_t) r^n_{t+1}$. Thus, firms' stock returns covary with R&D investment and physical investment in the same way as their physical investment returns covary. The implication is that stock returns are increasing in R&D investment but decreasing in physical investment.

2.6.2. Value versus growth

Value firms and growth firms have different expected stock returns because they have different levels of technological progress embodied in physical capital in the model. Book equity is identified as physical capital in the

[13] Cochrane (1997) considers aggregate equity returns, whereas Liu, Whited, and Zhang (2009) investigate the cross-section of equity returns.

[14] The marginal product of physical capital is often interpreted as the productivity of physical capital in producing output.

Including the last term in the numerator $(1-\delta_k)q^k_{t+1}$, three effects affect the expected physical investment return. But q^k_{t+1} is only a function of physical investment and R&D capital at $t+1$, (i_{t+1}, n_{t+2}), which implies that the effect of physical investment and R&D investment at t, (i_t, o_t), on q^k_{t+1} is secondary. Therefore, in analysis I focus on the marginal product of physical capital $\alpha y_{t+1}/k_{t+1}$ and the marginal cost of physical investment q^k_t.

[15] The marginal product of physical capital is strictly concave in R&D investment. More precisely, $\partial[\alpha y_{t+1}/k_{t+1}]/\partial o_t = (\partial[\alpha y_{t+1}/k_{t+1}]/\partial n_{t+1})\partial n_{t+1}/\partial o_t = \alpha(1-\alpha)\theta^{1-\alpha}e^{x_{t+1}+z_{t+1}}(k_{t+1})^{\alpha-1}(n_{t+1})^{-\alpha} > 0$ given $0<\alpha<1$.

[16] The marginal product of physical capital is strictly decreasing in physical investment. More precisely, $\partial[\alpha y_{t+1}/k_{t+1}]/\partial i_t = (\partial[\alpha y_{t+1}/k_{t+1}]/\partial k_{t+1})\partial k_{t+1}/\partial i_t = \alpha(\alpha-1)e^{x_{t+1}+z_{t+1}}(k_{t+1})^{\alpha-2}(\theta n_{t+1})^{1-\alpha}\Phi_1[i_t, (1-\theta)n_{t+1}] < 0$ given $0<\alpha<1$.

[17] Taking the partial derivative of the marginal cost of physical investment with respect to physical investment and R&D investment, I have $\partial[q^k_t]/\partial i_t > 0$ and, by chain rule, $\partial[q^k_t]/\partial o_t = (\partial[q^k_t]/\partial n_{t+1})\partial n_{t+1}/\partial o_t < 0$.

[18] The marginal product of R&D capital is strictly concave in physical capital. More precisely, I have $\partial[(1-\alpha)y_{t+1}/n_{t+1}]/\partial i_t = (\partial[(1-\alpha)y_{t+1}/n_{t+1}]/\partial k_{t+1})\partial k_{t+1}/\partial i_t = \alpha(1-\alpha)\theta^{1-\alpha}e^{x_{t+1}+z_{t+1}}(k_{t+1})^{\alpha-1}(n_{t+1})^{-\alpha}\Phi_1[i_t, (1-\theta)n_{t+1}] > 0$ given $0<\alpha<1$.

Table 1
Parameter values under benchmark calibration.
This table presents the calibrated parameter values of the benchmark model.

Notation	Value	Description
Group I		
α	0.75	Share of physical capital in output production
θ	0.70	Proportion of intangible capital devoted to final product
δ_k	0.10	Annual rate of physical capital depreciation
δ_n	0.20	Annual rate of intangible capital depreciation
ρ_x	0.98^4	Persistence coefficient of aggregate productivity
σ_x	0.014	Conditional volatility of aggregate productivity
$\overline{x}$	-1.57	Long-run average of aggregate productivity
ρ_z	0.70	Persistence coefficient of firm-specific productivity
σ_z	0.30	Conditional volatility of firm-specific productivity
β	0.94	Time-preference coefficient
γ_0	28	Constant price of risk
γ_1	-300	Time-varying price of risk
Group II		
a	0.45	Weight on physical investment in physical capital production
A	0.46	Constant term in physical capital production
ρ	0.50	Determining elasticity between physical investment and intangible capital

model. From Eq. (24), market-to-book ratio is $q_t^k+\tilde{q}_t^n n_{t+1}/k_{t+1}$. The market value of physical capital $q_t^k k_{t+1}$ is much larger than the market value of intangible capital $\tilde{q}_t^n n_{t+1}$, which implies that $q_t^k \gg \tilde{q}_t^n(n_{t+1}/k_{t+1})$. As a result, there is an approximately monotonic mapping from market-to-book ratio to the marginal cost of physical investment q_t^k. Value firms with low market-to-book ratios have low q_t^k and, therefore, have high expected physical investment returns. Growth firms with high market-to-book ratios have high q_t^k, which means they earn low expected physical investment returns. Because physical investment returns are dominant in stock returns, value firms earn high expected stock returns and growth firms earn low expected stock returns.

3. Main findings

This section presents the main findings of the paper.

3.1. Calibration

I divide the parameters of the benchmark model into two groups and then calibrate their annual values. Table 1 summarizes these values. Table 2 reports the model-implied moments and the data moments. (See Appendix B for data construction.)

The first group (Group I in Table 1) contains parameters that can be restricted by empirical research or quantitative studies: The share of physical capital is 0.75, estimated using National Income and Product Account (NIPA) data. (See Appendix C for estimation details.) The average proportion of R&D capital devoted to new product, θ, is set at 70% following the estimate in Cohen and Klepper (1996).[19] The physical capital depreciation rate $\delta_k=10\%$ is from Jermann (1998); the intangible capital depreciation rate δ_n is set at 20%; and persistence ρ_x and conditional volatility σ_x of aggregate productivity are from King and Rebelo (1999)—$\rho_x=0.98^4$, and $\sigma_x=0.007*2=0.014$.[20] The long-run average level of aggregate productivity, $\overline{x}$, is a scaling variable. I set the average long-run R&D capital in the economy at one,

Table 2
Key moments.
This table reports unconditional moments generated from the simulated data and the real data. I simulate one hundred artificial panels, each of which has three thousand firms, and each firm has one thousand annual observations. I report the cross-simulation averaged annual moments. The data moments of the annual average physical investment to asset ratio (physical investment scaled by physical capital in simulated data) and average market-to-book ratio are from Hennessy and Whited (2005). The other data moments are estimated from a sample from 1975 to 2009.

Moment	Data	Model
Annual average risk-free rate	1.89	1.65
Annual volatility of risk-free rate	2.21	3.20
Average annual Sharpe ratio	0.44	0.32
Average annual investment to asset ratio	0.15	0.11
Average market-to-book ratio	1.50	1.97
Annual research and development to physical investment ratio	0.51	0.62

[19] For a robustness check, I vary the parameter θ by setting $\theta=\{55\%,45\%\}$. I find that the qualitative implications of the model, (*footnote continued*) i.e., the positive relation between R&D investment and expected returns, do not change, but the quantitative implications vary with different values of θ. In particular, the premium associated with R&D investment is positively associated with the share of productivity increasing innovation, $1-\theta$.

[20] There is no agreement on the depreciation rate for R&D capital. However, the general agreement is that R&D capital depreciates faster than physical capital. I choose to use an annual rate of 20%, consistent with Chan, Lakonishok, and Sougiannis (2001). The calibration results are not sensitive to the depreciation rate of R&D capital.

which implies that the long-run average of aggregate productivity $\bar{x} = -1.57$. To calibrate persistence ρ_z and conditional volatility σ_z of firm-specific productivity, I restrict these two parameters using their implications on the degree of dispersion in the cross-sectional distribution of firms' stock return volatilities. Thus, $\rho_z = 0.70$ and $\sigma_z = 0.30$, which implies an average annual volatility of individual stock returns of 24.4%, approximately the average of 25% reported by Campbell, Lettau, Malkiel, and Yu (2001) and 32% reported by Vuolteenaho (2001).

Following Zhang (2005), I pin down the three parameters governing the stochastic discount factor, β, γ_0, and γ_1, to match three aggregate return moments: the average real interest rate, the volatility of the real interest rate, and the average annual Sharpe ratio. This procedure yields $\beta = 0.94$, $\gamma_0 = 28$, and $\gamma_1 = -300$, which generate an average annual real interest rate of 1.65%, an annual volatility of real interest rate of 3.2%, and an average annual Sharpe ratio of 0.32. Those values are close to the values obtained during the last 35 years (1975–2009) of data.

Prior studies provide only limited guidance for the calibration of the second group of parameters (Group II in Table 1). These parameters are a, the weight on physical investment in $\Phi[i_t,(1-\theta)n_{t+1}]$; A, the constant term in $\Phi[i_t,(1-\theta)n_{t+1}]$; and ρ, determining the elasticity of substitution between physical investment and intangible capital in $\Phi[i_t,(1-\theta)n_{t+1}]$. I pin down these three parameters to match three moments: the average annual rate of physical investment, the average annual market-to-book ratio, and the average annual R&D investment to physical investment ratio. This procedure yields $a = 0.45$, $A = 0.46$, and $\rho = 0.5$. The calibrated mean values of the physical investment rate and market-to-book ratio in the model are 0.11 and 1.97, respectively, close to the values of 0.15 and 1.50 reported by Hennessy and Whited (2005). The average ratio of R&D investment to physical investment is 0.62 in the model, close to the value of 0.51 in the data. In sum, the calibrated parameter values seem to be a reasonable representation of reality.

3.2. *Properties of model solutions*

In this subsection, I investigate the qualitative properties of the key variables in the model.

3.2.1. *Marginal cost of investments*

The formulation of the production function and the evolution of new physical capital have the following implications for the behavior of the marginal cost of physical investment and the effective marginal cost of R&D investment.

3.2.1.1. Marginal cost of physical investment. The critical variable in the model is q_t^k, the equilibrium marginal cost of physical investment. Panels A and B in Fig. 1 plot the numerical examples of q_t^k as functions of physical

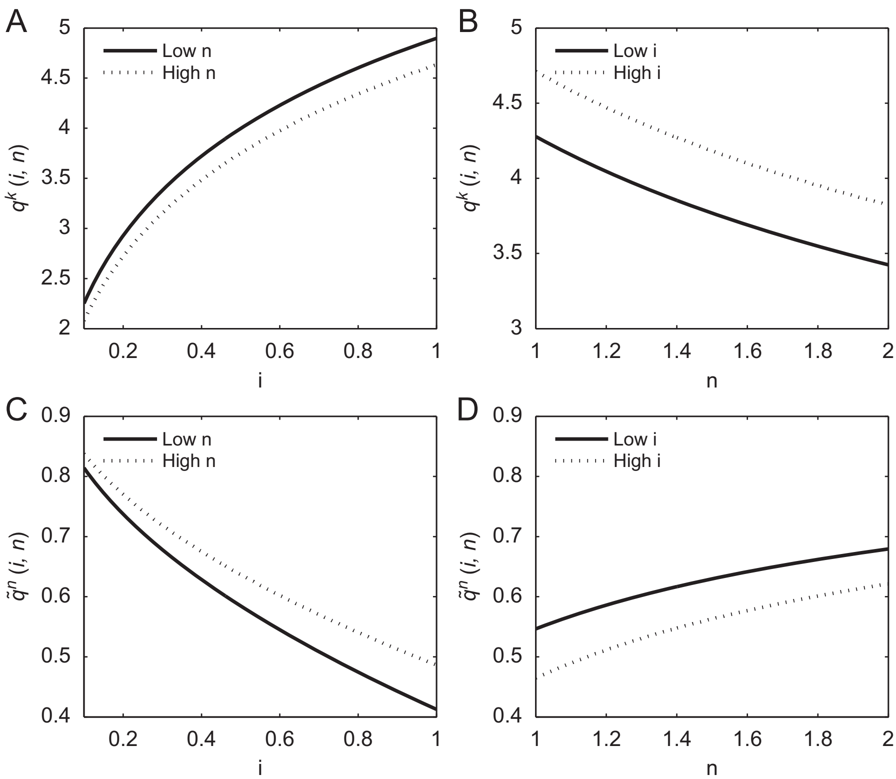

Fig. 1. Marginal cost of physical investment and the effective marginal cost of research and development (R&D) investment. This figure plots the marginal cost of physical investment q^k and the effective marginal cost of R&D investment $\tilde{q}^n$ as a function of physical investment i and intangible capital n. In Panels A and C, I plot q^k and $\tilde{q}^n$ against i. I then plot q^k and $\tilde{q}^n$ against n in Panels B and D.

investment i_t and intangible capital n_{t+1}. In Panel A, I plot q_t^k against physical investment i_t in two curves, each of which corresponds to one value of intangible capital n_{t+1}. In Panel B, I plot q_t^k against intangible capital n_{t+1} in two curves, each of which corresponds to one value of physical investment i_t. The marginal cost of physical investment q_t^k is increasing in physical investment i_t due to the diminishing marginal returns of $\Phi[i_t,(1-\theta)n_{t+1}]$ in i_t.[21] It is decreasing in intangible capital n_{t+1} because current technological progress makes new capital production more efficient and less expensive.[22]

q_t^k is concave in physical investment i_t, i.e., the elasticity of Tobin's Q of physical capital with respect to investment is positive but less than one. Compared with the standard one-capital model with quadratic adjustment costs (e.g., Hayashi, 1982) in which the Tobin's Q of physical capital is linear in investment, the two-capital benchmark model in the paper corresponds to a one-capital model with the curvature of adjustment cost function lower than two.[23]

3.2.1.2. Effective marginal cost of R&D investment. Panels C and D in Fig. 1 plot the numerical examples of the effective marginal cost of R&D investment $\tilde{q}_t^n$ as functions of physical investment i_t and intangible capital n_{t+1}. In Panel C, I plot $\tilde{q}_t^n$ against physical investment i_t in two curves, each of which corresponds to one value of intangible capital n_{t+1}. In Panel D, I plot $\tilde{q}_t^n$ against intangible capital n_{t+1} in two curves, each of which corresponds to one value of physical investment i_t. The effective marginal cost of R&D investment $\tilde{q}_t^n$ is decreasing in physical investment because the term of indirect benefit $(1-\theta)q_t^k\Phi_2[i_t,(1-\theta)n_{t+1}]$ is increasing in physical investment. The effective marginal cost of R&D investment $\tilde{q}_t^n$ is increasing in R&D capital due to the concavity of $\Phi[i_t,(1-\theta)n_{t+1}]$ in n_{t+1}.

3.3. Empirical predictions

Here, the quantitative implications concerning the cross-section of returns in the model are investigated. I show that a neoclassical model with endogenous technological progress driven by R&D investment is capable of simultaneously generating a positive relation between R&D investment and the subsequent average of stock returns and a negative relation between physical investment and the subsequent average of stock returns. The model also generates a positive relation between the book-to-market ratio and the subsequent average of stock returns.

The design of the quantitative experiment is as follows. I simulate one hundred panels, each with three thousand firms. Each firm has one thousand annual observations. The empirical procedure on each artificial sample is implemented and the cross-simulation results are reported. I then compare, when possible, the model moments with those in the data.

3.3.1. R&D investment and stock returns

I now investigate the empirical predictions of the model on the cross-section of stock returns and R&D investment. I focus on the work of Chan, Lakonishok, and Sougiannis (2001) and Li (forthcoming), who find a positive relation between R&D intensity and the subsequent average of stock returns.[24] Chan, Lakonishok, and Sougiannis (2001) interpret their results as indicating that investors are overly pessimistic about R&D firms' prospects. Li (forthcoming) attributes her results to the fact that R&D firms are more likely to be financially constrained. I show that a neoclassical model without investor irrationality or financing frictions can quantitatively replicate their evidence.

I follow Chan, Lakonishok, and Sougiannis (2001) in constructing five equal-weighted R&D portfolios for each simulated panel. The market value of equity in the model is defined as the ex dividend stock price. I sort all firms into five portfolios based on the firms' ratio of R&D investment to market value of equity, o_{t-1}/p_{t-1}^s, and the ratio of R&D investment to physical investment, o_{t-1}/i_{t-1}, in ascending order as of the beginning of year t. I then calculate the equal-weighted annual average excess returns for each R&D investment portfolio. I construct an R&D investment spread portfolio long in the high R&D intensity $(o_{t-1}/p_{t-1}^s, o_{t-1}/i_{t-1})$ portfolio and short in the low R&D intensity $(o_{t-1}/p_{t-1}^s, o_{t-1}/i_{t-1})$ portfolio. I repeat the entire simulation one hundred times and report the cross-simulation averages of the summary statistics in Table 3.

As Table 3 shows, consistent with Chan, Lakonishok, and Sougiannis (2001) and Li (forthcoming), firms with high R&D intensity, o_{t-1}/p_{t-1}^s (o_{t-1}/i_{t-1}), earn higher average excess returns than firms with low R&D intensity. The model generates a reliable R&D investment spread, which is 7.21% (7.52%) per annum for portfolios sorted on o_{t-1}/p_{t-1}^s and o_{t-1}/i_{t-1}, respectively. This R&D investment to physical investment spread is close to that shown in the data, 7.35%, but the R&D investment to equity spread falls short in magnitude compared with the data, 17.83%.

3.3.2. Physical investment and stock returns

I now investigate the empirical predictions of the model for the cross-section of stock returns and physical investment. I focus on Xing (2008), who finds that physical investment contains information similar to the

[21] In the model, $q_t^k = 1/\Phi_1[i_t,(1-\theta)n_{t+1}]$. Because $\Phi_{11}[i_t,(1-\theta)n_{t+1}] < 0, \partial q_t^k/\partial i_t > 0$.

[22] To see why this is the case, $q_t^k = 1/\Phi_1[i_t,(1-\theta)n_{t+1}]$. Because $\Phi_{12}[i_t,(1-\theta)n_{t+1}] > 0, \partial q_t^k/\partial n_{t+1} < 0$.

[23] Furthermore, I investigate the quantitative properties of Tobin's Q for physical capital in the two-capital and one-capital models. I focus on the volatility and persistence of Tobin's Q. I find that in the one-capital model the persistence and volatility of Tobin's Q for physical capital decrease in the curvature of the adjustment costs function. Intuitively, the higher the curvature, the less volatile the investment, and hence the smoother the Tobin's Q for physical capital. In the two-capital model, volatility and persistence are close to those of the one-capital model, with the curvature of the convex adjustment costs function smaller than two.

[24] Chan, Lakonishok, and Sougiannis (2001) and Li (forthcoming) use R&D investment scaled by the market value of equity and R&D investment to the physical investment ratio as R&D intensity, respectively.

Table 3
Stock returns of research and development (R&D) investment portfolios.
This table reports the excess stock returns of five portfolios sorted on R&D intensity where the R&D intensity is measured as the ratio of R&D investment to the market value of equity and R&D investment to the physical investment ratio, respectively. For June of year t from 1975 to 2009, I rank all firms based on the R&D intensity into five equal-numbered portfolios. I compute the subsequent annual equal-weighted returns from July of year t to June of year $t+1$ and reform the portfolios in June of year $t+1$. I simulate one hundred artificial panels, each of which has three thousand firms, and each firm has one thousand annual observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annualized returns in percentages. In the first column, R&D refers to R&D investment, physical refers to physical investment, and mkt refers to the market value of equity.

	Data						Model					
	Low	2	3	4	High	High–low	Low	2	3	4	High	High–low
R&D/mkt	6.38	7.93	11.66	16.94	24.21	17.83	2.34	4.63	5.16	6.69	9.55	7.21
R&D/physical	9.66	11.56	13.08	15.61	17.01	7.35	2.01	4.75	5.22	6.87	9.53	7.52

book-to-market ratio in explaining the value effect and that firms with a higher rate of physical investment earn lower average subsequent stock returns.

I follow Xing (2008) in constructing ten (both value-weighted and equal-weighted) portfolios sorted on the physical investment. I sort all firms into ten portfolios based on firms' rate of physical investment, i_{t-1}/k_{t-1}, in ascending order as of the beginning of year t. I construct a physical investment spread portfolio long in the low i_{t-1}/k_{t-1} portfolio and short in the high i_{t-1}/k_{t-1} portfolio, for each simulated panel. Table 4 reports the average excess stock returns of ten portfolios sorted on physical investment. Consistent with Xing (2008), firms with low i_{t-1}/k_{t-1} on average earn higher stock returns than firms with high i_{t-1}/k_{t-1}. The model-implied average value-weighted (equal-weighted) physical investment spread is 8.24% (8.54%) per annum, close to that in the data, 7.75% (11.22%).

3.3.3. *Abnormal physical investment and stock returns*

I now investigate the empirical predictions of the model for the cross-section of stock returns and abnormal physical investment. I focus on Titman, Wei, and Xie (2004), who find that firms with higher abnormal physical investment, defined as $CI_{t-1}=CE_{t-1}/((CE_{t-2}+CE_{t-3}+CE_{t-4})/3)-1$ in the portfolio formation year t, earn lower subsequent average stock returns after controlling for size, book-to-market ratio, and momentum (prior year return), where CE_{t-1} is the physical capital expenditure scaled by sales during year $t-1$. Titman, Wei, and Xie (2004) attribute their findings to investors' underreacting to the overinvestment behavior of empire-building managers. I show that a neoclassical model without investor irrationality can quantitatively replicate their evidence.[25]

I measure CE_{t-1} in the model as the physical investment to output ratio, i_{t-1}/y_{t-1}. The last three-year moving-average physical capital expenditure in the denominator of CI_{t-1} is used to proxy for firms' benchmark physical investment. I sort all firms into quintiles based on CI_{t-1} in ascending order as of the beginning of year t. I construct a CI-spread portfolio long in the low CI portfolio and short in the high CI portfolio, for each simulated panel.

Table 4
Stock returns of physical investment portfolios.
This table reports the value-weighted (VW) and equal-weighted (EW) excess stock returns of physical investment portfolios. Each year in June, firms are sorted into ten deciles by their previous fiscal year physical investment to asset ratio (ratio of physical investment to physical capital in simulated data), and I compute the value-weighted and equal-weighted returns on each decile portfolio. The low–high variable is the return difference between the lowest physical investment decile and the highest physical investment decile. The sample period is from 1975 to 2009. I simulate one hundred artificial panels, each of which has three thousand firms, and each firm has one thousand annual observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annualized returns in percentages.

	VW		EW	
	Data	Model	Data	Model
Low	10.45	10.37	17.25	10.42
2	10.28	8.46	14.72	8.46
3	8.46	7.75	14.08	7.69
4	9.83	6.53	13.73	6.44
5	9.88	6.00	14.75	5.99
6	7.75	5.76	13.53	5.67
7	8.06	5.65	12.24	5.61
8	6.74	3.53	11.55	3.57
9	5.14	3.03	10.71	3.03
High	2.7	2.13	6.03	1.88
Low–high	7.75	8.24	11.22	8.54

I calculate the value-weighted annual excess returns for each CI portfolio. Following Titman, Wei, and Xie (2004), I measure excess returns relative to benchmarks constructed to have similar firm characteristics such as size, book-to-market, and momentum. (See Appendix C for details about the empirical procedure.) Table 5 reports the average excess stock returns of five portfolios sorted on abnormal physical investment, CI. Consistent with the findings of Titman, Wei, and Xie (2004), firms with low CI earn higher average excess stock returns than firms with high CI. The model-implied average CI spread is 1.56% per annum. This spread is close to that shown in the data, 2.02%.

In sum, the benchmark model can simultaneously generate a positive covariation between R&D investment and future average stock returns and a negative covariation

[25] Li, Livdan, and Zhang (2009) also generate similar quantitative results, but with a different model.

Table 5
Excess stock returns of abnormal physical investment portfolios.

This table reports the excess stock returns of five portfolios ranking by abnormal physical investment, $CI_{t-1} = (CE_{t-1}/(CE_{t-2}+CE_{t-3}+CE_{t-4})/3-1)$ in the portfolio formation year t, where CE_{t-1} is the physical capital expenditure scaled by sales during year $t-1$. Data are the annualized returns from Titman, Wei, and Xie (2004). In each year t in the simulated panel, I rank firms based on abnormal physical investment, CI, into five equal-numbered portfolios. I compute the subsequent annual value-weighted returns from year t to year $t+1$ and reform the portfolios in year $t+1$. The excess return is the difference between each individual stock's return and the return of its matching portfolio by its size, book-to-market, and prior-year-return ranks. To form matching portfolios, I sort all firms each year into one of 125 size, book-to-market, and prior-year-return portfolios. See Appendix C for construction details of the benchmark. The low-high variable is the return difference between the lowest abnormal physical investment quintile and the highest abnormal physical investment quintile. I simulate one hundred artificial panels, each of which has three thousand firms, and each firm has one thousand annual observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annualized returns in percentages. In the first column, abnormal physical refers to abnormal physical investment.

	Data						Model					
	Low	2	3	4	High	Low–high	Low	2	3	4	High	Low–high
Abnormal physical	0.50	1.00	0.66	−1.00	−1.52	2.02	0.83	0.51	−0.26	−0.43	−0.73	1.56

between physical investment and future average stock returns.

3.3.4. The value premium

Here, I explore the relation between endogenous technological progress and the value premium.

First, I investigate whether the model can generate a positive relation between the book-to-market ratio and expected stock returns. I construct ten value-weighted and equal-weighted book-to-market portfolios. The book value of a firm in the model is identified as its physical capital stock. I sort all firms into ten portfolios based on firms' book-to-market ratio, k_{t-1}/p^s_{t-1}, in ascending order as of the beginning of year t. I construct a value-spread portfolio long in the high book-to-market portfolio and short in the low book-to-market portfolio for each simulated panel. Table 6 reports the average stock returns of ten portfolios sorted by the book-to-market ratio. Consistent with the findings of Fama and French (1992, 1993), firms with low book-to-market ratios earn lower stock returns on average than do firms with high book-to-market ratios. The model-implied average value-weighted (equal-weighted) value spread is 7.78% (8.06%) per annum. This value-weighted spread is close to that shown in the data, 8.83%, but the equal-weighted spread falls short in magnitude compared with the data, 18.77%.

Table 6
Stock returns of book-to-market portfolios.

This table reports the value-weighted (VW) and equal-weighted (EW) excess stock returns of ten portfolios sorted on the book-to-market ratio. Data are from Ken French's website. The high-low variable is the return difference between the highest book-to-market decile and the lowest book-to-market decile. The sample period is from 1975 to 2009. I simulate one hundred artificial panels, each of which has three thousand firms, and each firm has one thousand annual observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annualized returns in percentages.

	VW		EW	
	Data	Model	Data	Model
Low	5.86	1.14	5.22	1.14
2	8.25	2.50	10.79	2.28
3	8.92	3.66	12.34	3.67
4	9.93	5.86	14.03	5.84
5	8.86	5.90	14.78	5.92
6	9.15	5.95	15.64	5.91
7	10.40	6.54	16.21	6.40
8	10.09	7.34	16.78	7.27
9	11.89	8.17	19.56	8.19
High	14.69	10.28	23.99	10.34
High–low	8.83	7.78	18.77	8.06

3.4. Causality

I now focus on causal relations that explains why R&D investment positively forecasts average stock returns while physical investment negatively forecasts average stock returns in the model. I also investigate the relation between endogenous technological progress and the value premium.

3.4.1. Investment returns and investment

First, I examine the covariations between investment returns (both R&D and physical) and investment. In Panels A and B of Table 7, I report the simulated average physical investment returns and R&D investment returns for five portfolios sorted on R&D intensity and rate of physical investment, respectively. The expected return on physical investment is negatively related to physical investment but positively related to R&D investment. This is because R&D investment increases the marginal product of physical capital, and R&D (physical) investment decreases (increases) the marginal cost of physical investment, which is negatively related to the expected physical investment returns. The expected return on R&D investment covaries positively with physical investment and covaries negatively with R&D investment. That is because the expected marginal product of R&D capital (the effective marginal cost of R&D investment) is decreasing (increasing) in R&D investment but increasing (decreasing) in physical investment. So investments (both R&D and physical) covary with the expected physical investment return and R&D investment return in opposite ways. That leads to two countervailing effects on the predictability of investments on future average stock returns. The weights on R&D investment return and

Table 7
Weights on investment returns and investment returns.

This table reports equal-weighted simulated excess research and development (R&D) investment returns, excess physical investment returns, and their respective weights in the stock returns of five portfolios sorted on R&D intensity (measured as R&D investment scaled by market value of equity and R&D investment to physical investment ratio) and the rate of physical investment. I simulate one hundred artificial panels, each of which has three thousand firms, and each firm has one thousand annual observations. All returns are simple, annualized returns in percentages. In the first column, R&D refers to R&D investment, mkt refers to the market value of equity, and physical refers to physical investment.

	Low	2	3	4	High
Panel A: Physical investment returns					
R&D/mkt	1.89	4.26	5.71	8.28	13.31
R&D/physical	0.65	4.37	5.63	8.00	14.80
Physical/physical capital	16.82	9.98	8.10	6.42	1.79
Panel B: R&D investment returns					
R&D/mkt	7.73	7.09	6.87	6.34	6.19
R&D/physical	8.26	8.18	7.00	6.18	4.70
Physical/physical capital	3.99	8.05	8.26	8.57	8.75
Panel C: Weights on physical investment returns					
R&D/mkt	0.72	0.71	0.69	0.66	0.61
R&D/physical	0.73	0.71	0.69	0.66	0.60
Physical/physical capital	0.59	0.65	0.67	0.70	0.71
Panel D: Weights on R&D investment returns					
R&D/mkt	0.28	0.29	0.31	0.34	0.39
R&D/physical	0.27	0.29	0.31	0.34	0.40
Physical/physical capital	0.41	0.35	0.33	0.30	0.29

physical investment return need to be examined to determine which effect dominates.

3.4.2. *Weights on investment returns*

Panels C and D in Table 7 report the simulated average weights on the physical investment returns and R&D investment returns for five portfolios sorted on R&D intensity and rate of physical investment, respectively. The weight on the physical investment return $q_t^k k_{t+1}/p_t^s$ is much greater than the weight on the R&D investment return $\tilde{q}_t^n n_{t+1}/p_t^s$. This is because physical capital production involves both intangible capital and physical capital, and the share of intangible capital in the output production is smaller than that of the tangible capital. The difference in weights between the physical investment return and the R&D investment return implies that the physical investment return, together with its weight $(q_t^k k_{t+1}/p_t^s) r_{t+1}^k$, dominates in stock returns. That is why R&D investment positively forecasts average future stock returns while physical investment negatively forecasts average future stock returns.

3.4.3. *Endogenous technological progress and the value premium*

In the model, value firms invest less in intangible capital than do growth firms, so value firms do not gain as much from technological progress in increasing the productivity of physical capital as growth firms do. When a recession occurs, value firms are stuck with excessive physical capital and do not have much endogenous technological progress to upgrade the efficiency of physical capital. They are, therefore, more risky than growth firms, given that the price of risk is high during economic downturns. This interaction between endogenous technological progress and physical capital reinforces the mechanism emphasized in Zhang (2005), who demonstrates that costly reversibility of physical capital is a key mechanism driving the value premium.

3.5. *Discussion*

I discuss the implications of the key assumptions in the model.

3.5.1. *Product innovation versus productivity increasing innovation*

The crucial channel in the model in generating a positive relation between R&D investment and average stock returns is productivity increasing innovation. This is because, on the one hand, if all R&D investment is devoted to creating new products ($\theta = 1$), the model reduces to a standard one that predicts that both R&D investment and physical investment forecast expected stock returns in the same way, which is counterfactual. On the other hand, if all R&D investment is dedicated to increasing productivity of physical investment ($\theta = 0$) and output production is linear in physical capital, the model can still simultaneously explain the relations of R&D investment and physical investment with average stock returns. In addition, the model-implied R&D premium is decreasing with the share of product innovation, consistent with Lev and Sougiannis (1999), who show that the R&D investment premium is more pronounced in industries with less product innovation.

3.5.2. *Stock or flow*

In the model, n_{t+1} instead of o_t appears in the accumulation equation for k_{t+1} because n_{t+1} is the stock of ideas and knowledge while o_t is the flow.

If the flow o_t appears in the accumulation equation for k_{t+1}, the central qualitative prediction does not change, i.e., R&D investment still positively forecasts expected stock returns. Quantitatively, the positive correlation between R&D investment and expected stock returns is stronger when the flow o_t enters the accumulation process. The reason is as follows.

If R&D investment o_t directly enters the physical capital accumulation process, then

$$k_{t+1} = (1-\delta_k)k_t + \Phi[i_t,(1-\theta)o_t]. \tag{34}$$

To compare with the specification in the model when the stock n_{t+1} enters the accumulation, Eq. (5) is rewritten by substituting n_{t+1} with Eq. (4), which gives

$$k_{t+1} = (1-\delta_k)k_t + \Phi[i_t,(1-\theta)(o_t + (1-\delta_n)n_t)]. \tag{35}$$

Qualitatively, R&D investment reduces the marginal cost of physical investment in producing new physical capital in both Eqs. (34) and (35). However, when n_{t+1} enters in the accumulation equation in Eq. (35), both R&D investment o_t and last-period R&D capital net of depreciation $(1-\delta_n)n_t$ appear in the accumulation process. Because o_t and n_t are negatively correlated, the effect of o_t on the marginal cost of physical investment is weaker in Eq. (35) than it is in Eq. (34). Therefore, the predictive power of R&D investment for the expected stock returns is stronger when R&D investment o_t directly enters accumulation dynamics for k_{t+1}.

3.5.3. Labor

Notably, labor does not enter the model. First, if labor enters production as a flexible factor and gets paid on the marginal product of labor as in the standard neoclassical model, labor (hiring) will not have explanatory power for the expected stock returns, as has been shown in Bazdresch, Belo, and Lin (2009). If production function remains constant returns to scale in capital and labor, the qualitative prediction of the model does not change, i.e., the stock return is still a weighted average of the physical investment return and the R&D investment return. Quantitatively, with labor being flexible, the magnitude of expected stock returns drops because firms can use labor to smooth productivity shocks (Boldrin, Christiano, and Fisher, 2001).

Second, if labor is quasi-fixed and reproducible (the same as capital), there also is an intertemporal return for labor hiring. Hence, stock return is a weighted average of the physical investment return, the R&D investment return, and the hiring return. Bazdresch, Belo, and Lin (2009) show that labor hiring negatively forecasts expected stock returns empirically, and the negative relation can be captured with a neoclassical model augmented with labor adjustment costs. Based on their findings, I expect that as long as R&D capital increases the productivity in producing new capital in the model, the qualitative predictions (i.e., R&D investment positively forecasting expected stock returns) hold because physical investment and labor hiring predict the expected stock returns in the same direction. But, this model is beyond the scope of this paper. I leave it as an interesting topic for future research.

4. Concluding remarks

Following Cochrane (1991, 1996), I show that a neoclassical model with endogenous technological progress driven by R&D investment can explain a number of empirical regularities in the cross-section of stock returns. Most notably, technological progress endogenously driven by R&D investment raises the expected marginal benefit of physical capital and reduces the marginal cost of physical investment, causing the expected returns in physical investment increasing in R&D investment. The expected physical investment return is decreasing in physical investment due to the diminishing marginal returns of physical capital production. In the model, the weight on the physical investment return dominates the weight on the R&D investment return. Thus, the model simultaneously explains why R&D investment—intensive firms earn high average stock returns while physical investment—intensive firms earn low average stock returns. The positive predictability of R&D investment on expected stock returns, interpreted by Chan, Lakonishok, and Sougiannis (2001) as excessive pessimism, is in principle consistent with rational expectations. The model also explains why value firms are more risky than growth firms: Value firms invest less in R&D capital and, thus, do not have as much technological progress in upgrading the efficiency of the existing physical capital as growth firms do, especially during bad times.

Future research can proceed in several directions. Theoretically, a full-fledged general equilibrium model with Epstein-Zin preferences can link endogenous technological progress to long-run consumption risk. The neoclassical framework in the model can also be extended to link asset prices with other types of intangible capital, e.g., human capital and organizational capital. Empirically, the correlation between human capital, organizational capital, and physical capital and their relations with the cross-section of stock returns is worth investigating further.

Appendix A. Proof

I first show $p_t^s = q_t^k k_{t+1} + \tilde{q}_t^n n_{t+1}$. The production function is constant returns to scale:

$$y_t = e^{x_t+z_t}(k_t)^\alpha(\theta n_t)^{1-\alpha}. \tag{36}$$

Transversality conditions for k_{t+1+j} and n_{t+1+j} are

$$\lim_{j\to\infty} \mathbb{E}_t M_{t,t+j} q_{t+j}^k k_{t+1+j} = 0 \tag{37}$$

and

$$\lim_{j\to\infty} \mathbb{E}_t M_{t,t+j} n_{t+1+j} = 0. \tag{38}$$

Define firms' cum-dividend market value as

$$v(k_t,n_t,x_t,z_t) \equiv p_t^s + d_t. \tag{39}$$

The dividend is given by

$$\begin{aligned} d_t &= y_t - i_t - o_t \\ &= e^{x_t+z_t}(k_t)^\alpha(\theta n_t)^{1-\alpha} - i_t - o_t. \end{aligned} \tag{40}$$

Combining Eqs. (39) and (40), I get

$$v(k_t,n_t,x_t,z_t) = p_t^s + e^{x_t+z_t}(k_t)^\alpha(\theta n_t)^{1-\alpha} - i_t - o_t. \tag{41}$$

The physical capital accumulation process in Eq. (5) can be rewritten as

$$\begin{aligned} k_{t+1} = {} & (1-\delta_k)k_t + i_t \Phi_1[i_t,(1-\theta)n_{t+1}] \\ & + (1-\theta)n_{t+1}\Phi_2[i_t,(1-\theta)n_{t+1}], \end{aligned} \tag{42}$$

where Φ_1 and Φ_2 are partial derivatives of $\Phi[i_t,(1-\theta)n_{t+1}]$ with respect to the first and second argument. Expanding the value function in Eq. (16) and using Eq. (42), I get

$$\begin{aligned} & v(k_t,n_t,x_t,z_t) \\ & = \mathbb{E}_t \sum_{j=0}^{\infty} M_{t,t+j}\{(e^{x_{t+j}+z_{t+j}}(k_{t+j})^\alpha(\theta n_{t+j})^{1-\alpha} - i_{t+j} - o_{t+j}) \\ & \quad - q_{t+j}^k[k_{t+1+j} - (1-\delta_k)k_{t+j} - i_{t+j}\Phi_1 \\ & \quad - (1-\theta)n_{t+1+j}\Phi_2] - \tilde{q}_{t+j}^n[n_{t+1+j} - (1-\delta_n)n_{t+j} - o_{t+j}]\}. \end{aligned} \tag{43}$$

Recursively substituting Eqs. (4), (5), and (17)–(20), I find

$$\begin{aligned}v(k_t,n_t,x_t,z_t) &= e^{x_t+z_t}(k_t)^{\alpha}(\theta n_t)^{1-\alpha}+q_t^k(1-\delta_k)k_t+(1-\delta_n)n_t \\ &\quad -\lim_{j\to\infty}\mathbb{E}_t M_{t,t+j}q_{t+j}^k k_{t+1+j}-(1-\delta_n)\lim_{j\to\infty}\mathbb{E}_t M_{t,t+j}n_{t+1+j} \\ &= e^{x_t+z_t}(k_t)^{\alpha}(\theta n_t)^{1-\alpha}+q_t^k(1-\delta_k)k_t+(1-\delta_n)n_t. \end{aligned} \quad (44)$$

Combining with Eq. (41), I get

$$\begin{aligned} p_t^s+e^{x_t+z_t}(k_t)^{\alpha}(\theta n_t)^{1-\alpha}-i_t-o_t \\ = e^{x_t+z_t}(k_t)^{\alpha}(\theta n_t)^{1-\alpha}+q_t^k(1-\delta_k)k_t+(1-\delta_n)n_t. \end{aligned} \quad (45)$$

Rearranging and using Eq. (42) leads to

$$p_t^s = q_t^k k_{t+1}+[1-(1-\theta)q_t^k\Phi_2[i_t,(1-\theta)n_{t+1}]]n_{t+1} \quad (46)$$

$$= q_t^k k_{t+1}+\tilde{q}_t^n n_{t+1}. \quad (47)$$

Now I show $r_{t+1}^s = (q_t^k k_{t+1}/p_t^s)r_{t+1}^k+(\tilde{q}_t^n n_{t+1}/p_t^s)r_{t+1}^n$. Define the stock return as

$$r_{t+1}^s \equiv \frac{p_{t+1}^s+d_{t+1}}{p_t^s}. \quad (48)$$

Using Eq. (40) implies

$$r_{t+1}^s = \frac{p_{t+1}^s+e^{x_{t+1}+z_{t+1}}(k_{t+1})^{\alpha}(\theta n_{t+1})^{1-\alpha}-i_{t+1}-o_{t+1}}{p_t^s}. \quad (49)$$

Using Eq. (46) I find

$$r_{t+1}^s = \frac{q_{t+1}^k k_{t+2}+(1-(1-\theta)q_{t+1}^k\Phi_2)n_{t+2}+e^{x_{t+1}+z_{t+1}}(k_{t+1})^{\alpha}(\theta n_{t+1})^{1-\alpha}-i_{t+1}-o_{t+1}}{p_t^s}. \quad (50)$$

Because $\Phi[i_{t+1},(1-\theta)n_{t+2}]$ is constant returns to scale in $(i_{t+1},(1-\theta)n_{t+2})$, I get

$$\begin{aligned}\Phi[i_{t+1},(1-\theta)n_{t+2}] &= i_{t+1}\Phi_1[i_{t+1},(1-\theta)n_{t+2}] \\ &\quad +(1-\theta)n_{t+2}\Phi_2[i_{t+1},(1-\theta)n_{t+2}]. \end{aligned} \quad (51)$$

This implies

$$\begin{aligned} r_{t+1}^s &= \frac{q_{t+1}^k(1-\delta_k)k_{t+1}+(1-\delta_n)n_{t+1}+e^{x_{t+1}+z_{t+1}}(k_{t+1})^{\alpha}(\theta n_{t+1})^{1-\alpha}}{p_t^s} \\ &= \frac{q_t^k k_{t+1}\left[\dfrac{\alpha y_{t+1}/k_{t+1}+(1-\delta_k)q_{t+1}^k}{q_t^k}\right]}{p_t^s} \\ &\quad + \frac{\tilde{q}_t^n n_{t+1}\left[\dfrac{(1-\alpha)y_{t+1}/n_{t+1}+(1-\delta_n)}{\tilde{q}_t^n}\right]}{p_t^s} \\ &= \frac{q_t^k k_{t+1}}{p_t^s}r_{t+1}^k+\frac{\tilde{q}_t^n n_{t+1}}{p_t^s}r_{t+1}^n. \end{aligned} \quad (52)$$

Appendix B. Data construction

I use annual Center for Research in Security Prices (CRSP) value-weighted returns (1975–2009) from Ken French's website as stock market returns. The annual risk-free rate is from Ken French's website. Monthly stock returns are from CRSP. The size of a firm is its market capitalization in June, taken from CRSP. Compustat annual item CAPX is used for physical investment, i, and the net book value of property, plant, and equipment (annual item PPENT) is used for the net fixed assets, k. Compustat annual item XRD is used for R&D investment, o.

Appendix C. Empirical procedure

C.1. Estimating the aggregate production function

The aggregate production function is given by $y = e^x(k)^{\alpha}(\theta n)^{1-\alpha}$. I estimate α using data. Output y is GDP from NIPA Table 1.1.5, physical capital k is private nonresidential fixed assets from NIPA Table 4.1, and R&D capital n is net stock of private R&D assets from NIPA Table 3.4. The sample period is 1975–2002.

C.2. Calculating characteristic-adjusted excess returns for abnormal physical investment portfolios à la Titman, Wei, and Xie (2004)

To calculate the characteristic-adjusted excess returns of the physical investment portfolios in simulated data, I follow Titman, Wei, and Xie (2004). Specifically, I form 125 benchmark portfolios that capture these characteristics. Starting in year t, the universe of common stocks is sorted into five portfolios based on firm size at the end of year $t-1$. The breakpoints for size are obtained by sorting all firms into quintiles based on their size measures at the end of year $t-1$ in ascending order. The size of each firm is then compared with the breakpoints to decide which portfolio the firm belongs to. Firms in each size portfolio are further equally sorted into quintiles based on their book-to-market ratio at the end of year $t-1$. Finally, the firms in each of the 25 size and book-to-market portfolios are equally sorted into quintiles based on their prior year return. In all, I obtain 125 benchmark portfolios.

I calculate excess returns using these 125 characteristic-based benchmark portfolios. Each year, each stock is assigned to a benchmark portfolio according to its rank based on size, book-to-market, and prior year returns. The excess annual returns of a stock are calculated by subtracting the returns of the corresponding benchmark portfolio from the returns of this particular stock. The excess returns on individual stocks are then used to calculate the value-weighted excess annual returns on the test portfolios that are formed based on abnormal physical investment.

Appendix D. Numerical method

To solve the model numerically, I use the value function iteration procedure to solve the firm's maximization problem. The value function and the optimal decision rule are solved on a grid in a discrete state space. I specify a grid with one hundred points each for the physical capital and intangible capital, respectively, with upper bounds $\overline{k}$, $\overline{n}$ (large enough to be nonbinding at all times). The grids for physical capital and intangible capital stocks are constructed recursively, following McGrattan (1999), that is, $k_i = k_{i-1} + c_{k1}\exp(c_{k2}(i-2))$, where $i = 1,\ldots,100$ is the index of grids points and c_{k1} and c_{k2} are two constants chosen to provide the desired number of grid points and two upper bounds $\overline{k}$, $\overline{n}$, given two pre-specified lower bounds $\underline{k}$, $\underline{n}$. The advantage of this recursive construction is that more grid points are assigned around $\underline{k}$,$\underline{n}$, where the value function has most of its curvature.

The state variable x is defined on continuous state space, which has to be transformed into discrete state space. I use the method described in Rouwenhorst (1995) that works well when the persistence level is above 0.9. I use nine grid points for an x process and nine grid points for a z process. In all cases, the results are robust to finer grids as well. Once the discrete state space is available, the conditional expectation can be carried out simply as a matrix multiplication. Linear interpolation is used extensively to obtain optimal investments that do not lie directly on the grid points. Finally, I use a simple discrete, global search routine in maximizing problems.

References

Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60, 323–351.

Albuquerque, R., Wang, N., 2008. Agency conflicts, investment, and asset pricing. Journal of Finance 63, 1–40.

Bazdresch, S., Belo, F., Lin, X., 2009. Labor hiring, investment, and stock return predictability in the cross section. Unpublished working paper, University of Minnesota and London School of Economics and Political Science, Minneapolis and Saint Paul, MN, London, England.

Belo, F., 2010. Production-based measures of risk for asset pricing. Journal of Monetary Economics 57, 146–163.

Boldrin, M., Christiano, L., Fisher, J., 2001. Habit persistence, asset returns, and the business cycle. American Economic Review 91, 149–166.

Bolton, P., Chen, H., Wang, N., 2011. A unified theory of Tobin's q, corporate investment, financing, and risk management. Journal of Finance 66, 1545–1578.

Campbell, J.Y., Cochrane, J.H., 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. Journal of Political Economy 107, 205–251.

Campbell, J.Y., Lettau, M., Malkiel, B.G., Xu, Y., 2001. Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. Journal of Finance 56, 1–43.

Carlson, M., Fisher, A., Giammarino, R., 2004. Corporate investment and asset price dynamics: implications for the cross section of returns. Journal of Finance 59, 2577–2603.

Chan, L., Lakonishok, J., Sougiannis, T., 2001. The stock market valuation of research and development expenditures. Journal of Finance 56, 2431–2456.

Cochrane, J.H., 1991. Production-based asset pricing and the link between stock returns and economic fluctuations. Journal of Finance 46, 209–237.

Cochrane, J.H., 1996. A cross-sectional test of an investment-based asset pricing model. Journal of Political Economy 104, 572–621.

Cochrane, J.H., 1997. Where is the market going? Uncertain facts and novel theories. Economic Perspectives 6, 3–37.

Cochrane, J.H., 2001. Asset Pricing. Princeton University Press, Princeton, NJ.

Cohen, W.M., Klepper, S., 1996. Firm size and the nature of innovation with industries: the case of process and product R&D. Review of Economics and Statistics 78, 232–243.

Cooper, I., 2006. Asset pricing implications of nonconvex adjustment costs and irreversibility of investment. Journal of Finance 61, 139–170.

DeMarzo, P., Fishman, M., He, Z., Wang, N. Dynamic agency and the q theory of investment. Journal of Finance, forthcoming.

Dixit, A., Stiglitz, J., 1977. Monopolistic competition and optimum product variety. American Economic Review 67, 297–308.

Fama, E.F., French, K.R., 1992. The cross section of expected stock returns. Journal of Finance 47, 427–465.

Fama, E.F., French, K.R., 1993. Common risk factors in return of stocks and bonds. Journal of Financial Economics 33, 3–56.

Fisher, J., 2006. The dynamic effects of neutral and investment-specific technology shocks. Journal of Political Economy 114, 413–451.

Galeotti, M., Schiantarelli, F., 1991. Generalized Q models for investment. Review of Economics and Statistics 73, 383–392.

Gomes, J., Kogan, L., Zhang, L., 2003. Equilibrium cross section of returns. Journal of Political Economy 111, 693–732.

Greenwood, J., Hercowitz, Z., Krusell, P., 1997. Long-run implications of investment-specific technological change. American Economic Review 87, 342–362.

Greenwood, J., Hercowitz, Z., Krusell, P., 2000. The role of investment-specific technological change in the business cycle. European Economic Review 44, 91–115.

Hansen, L.P., Heaton J., Li, N., 2004. Intangible risk? Unpublished working paper, University of Chicago, Chicago, IL.

Hayashi, F., 1982. Tobin's marginal q and average q: a neoclassical interpretation. Econometrica 50, 213–224.

Hennessy, C.A., Whited, T.M., 2005. Debt dynamics. Journal of Finance 60, 1129–1165.

Hennessy, C.A., Whited, T.M., 2007. How costly is external financing? Evidence from a structural estimation. Journal of Finance 62, 1705–1745.

Hsu, P.-H., 2009. Technological innovations and aggregate risk premiums. Journal of Financial Economics 94, 264–279.

Huffman, G., 2007. Endogenous growth through investment-specific technological change. Review of Economic Dynamics 10, 615–645.

Jermann, U., 1998. Asset pricing in production economy. Journal of Monetary Economics 41, 257–275.

King, R.G., Rebelo, S.T., 1999. Resuscitating real business cycles. In: Taylor, J., Woodford, M. (Eds.), Handbook of MacroeconomicsNorth-Holland, Amsterdam, pp. 927–1007.

Lamont, O.A., 2000. Investment plans and stock returns. Journal of Finance 55, 2719–2745.

Lev, B., Sougiannis, T., 1999. Penetrating the book-to-market black box: the R&D effect. Journal of Business Finance and Accounting 26, 419–449.

Levin, R., Reiss, P., 1988. Cost-reducing and demand-creating R&D with spillovers. Rand Journal of Economics 19, 538–556.

Li, D. Financial constraints R&D investment, and stock returns. Review of Financial Studies, forthcoming.

Li, E.X.N., Livdan, D., Zhang, L., 2009. Anomalies. Review of Financial Studies 22, 4301–4334.

Lin, E., Saggi, K., 2002. Product differentiation, process R&D and the nature of market competition. European Economic Review 46, 201–211.

Liu, L.X., Whited, T., Zhang, L., 2009. Investment-based expected stock returns. Journal of Political Economy 117, 1105–1139.

Livdan, D., Sapriza, H., Zhang, L., 2009. Financially constrained stock returns. Journal of Finance 64, 1827–1862.

Mas-Colell, A., Whinston, M.D., Green, J.R., 1995. Microeconomic Theory. Oxford University Press, New York.

McGrattan, E., 1999. Application of weighted residual methods to dynamic economic models. In: Marimon, R., Scott, A. (Eds.), Computational Methods for the Study of Dynamic EconomiesOxford University Press, Oxford, England, pp. 114–142.

McGrattan, E., Prescott, E., 2005. Taxes, regulations, and the value of US and UK corporations. Review of Economic Studies 72, 767–796.

Merz, M., Yashiv, E., 2007. Labor and the market value of the firm. American Economic Review 97, 1419–1431.

Romer, P., 1990. Endogenous technological change. Journal of Policy Economy 98, 72–102.

Rouwenhorst, G., 1995. Asset pricing implications of equilibrium business cycle models. In: Cooley, T.F. (Ed.), Frontiers of Business Cycle ResearchPrinceton University Press, Princeton, NJ, pp. 294–330.

Titman, S., Wei, J., Xie, F., 2004. Capital investments and stock returns. Journal of Financial and Quantitative Analysis 39, 677–700.

Vuolteenaho, T., 2001. What drives the firm-level stock returns? Journal of Finance 57, 233–264.

Xing, Y., 2008. Interpreting the value effect through the Q-theory: an empirical investigation. Review of Financial Studies 21, 1767–1795.

Zhang, L., 2005. The value premium. Journal of Finance 60, 67–103.