\[ x_c[n] = A \cos(\hat{\omega}_0 n) = A \cos(2\pi \hat{f}_0 n) \]

\[ \hat{f}_0 = 0 \]

\[ \hat{f}_0 = \frac{1}{8}, \quad \hat{\omega}_0 = \frac{\pi}{4} \]

8 samples per cycle

\[ \hat{f}_0 = \frac{1}{4}, \quad \hat{\omega}_0 = \frac{\pi}{2} \]

4 samples per cycle

\[ \hat{f}_0 = \frac{1}{2}, \quad \hat{\omega}_0 = \pi \]

2 samples per cycle
4.1.4 The Sampling Theorem

Fig 4-3 - How frequently must we sample in order to retain enough information to reconstruct the original continuous-time signal from the samples?

Shannon Sampling Theorem:
A continuous-time signal $x(t)$ with frequencies no higher than $f_{max}$ can be reconstructed exactly from its samples $x[n] = x(nT_s)$ if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{max}$.

Audio signals (speech) $f_{max} \approx 3000$ Hz
TV signals $f_{max} \approx 5$ MHz

\[ f_s \geq 2f_{max} \]
\[ f_{max} < \frac{f_s}{2} \]  
Nyquist rate
CD's use $f_s = 44.1 \text{ KHz}$

$20 \text{ KHz} \rightarrow$ threshold of human hearing and perception

$h_{\text{max}} = 20 \text{ KHz} \leq \frac{f_s}{2} = 22 \text{ KHz}$

**Sinusoid signal:**

$$A \cos(2\pi ft + \phi)$$

$h_{\text{max}} = f$

$$f \leq \frac{f_s}{2} \quad f_s \geq 2f$$

**Minimum sampling rate**

$$f_s = 2f$$

Note: $\hat{f}$ of discrete sinusoid $= \frac{1}{2}$
Decibels

\[ x(t) \rightarrow \text{voltage or current signal} \]

\[ \text{dB (Decibels)} \rightarrow 20 \log_{10} x(t) \]

\[
\begin{align*}
\lim_{x \to 0^+} \log_{10} x &= -\infty \\
\log_{10} 1 &= \log_{10} 10 = 0 \\
\log_{10} 10 &= 1 \\
\log_{10} 100 &= 2
\end{align*}
\]
\[ x(t) \]

\[ 20 \log_{10} 100 = 40 \text{ dB} \]
\[ 20 \log_{10} 10 = 20 \text{ dB} \]

\[ x(t) \] dB

\[ 20 \log_{10} \text{Hz} = 20 \text{ dB} \] magnified

\[ 20 \log_{10} \text{Hz} = 20 \text{ dB} \] compressed
\[ b_i = 20 \log b \]
\[ a_i = 20 \log a \]
\[ b_i - a_i = 20 \log b - 20 \log a \]
\[ b_i - a_i = 20 \log \frac{b}{a} \]
\[ b = \frac{(b_i - a_i)}{20} \]
\[ \frac{b}{a} = 10 \]