Final Solution ECE2000 Autumn 2015

Show all relevant steps. Don't just write down the answers.

Show your work on these pages, attach additional pages if necessary.

- **Staple** all your work together
- Fill out the following section. **You will lose 5 points if you fail to provide these details**

Your Last Name_____ Your First Name_____

1. Lecture Student _____ or Recitation Student_____ (check one)
2. If Recitation then fill out the following
   Name of recitation instructor_____ Date/time of recitation_____
3. Your Lab Section/Group_____

Problem 1 (50 points): A circuit has four inputs C1, C2, X1 and X2. The inputs C1 and C2 behave as "control" inputs and X1 and X2 behave as "data" inputs. The circuit has one output Z. The circuit behaves as follows:

If C1 = 0 and C2 = 0 then Z = X1 . X2
If C1 = 0 and C2 = 1 then Z = X1 $\oplus$ X2
If C1 = 1 and C2 = 0 then Z = X1 + X2
If C1 = 1 and C2 = 1 then Z = X1 $\equiv$ X2

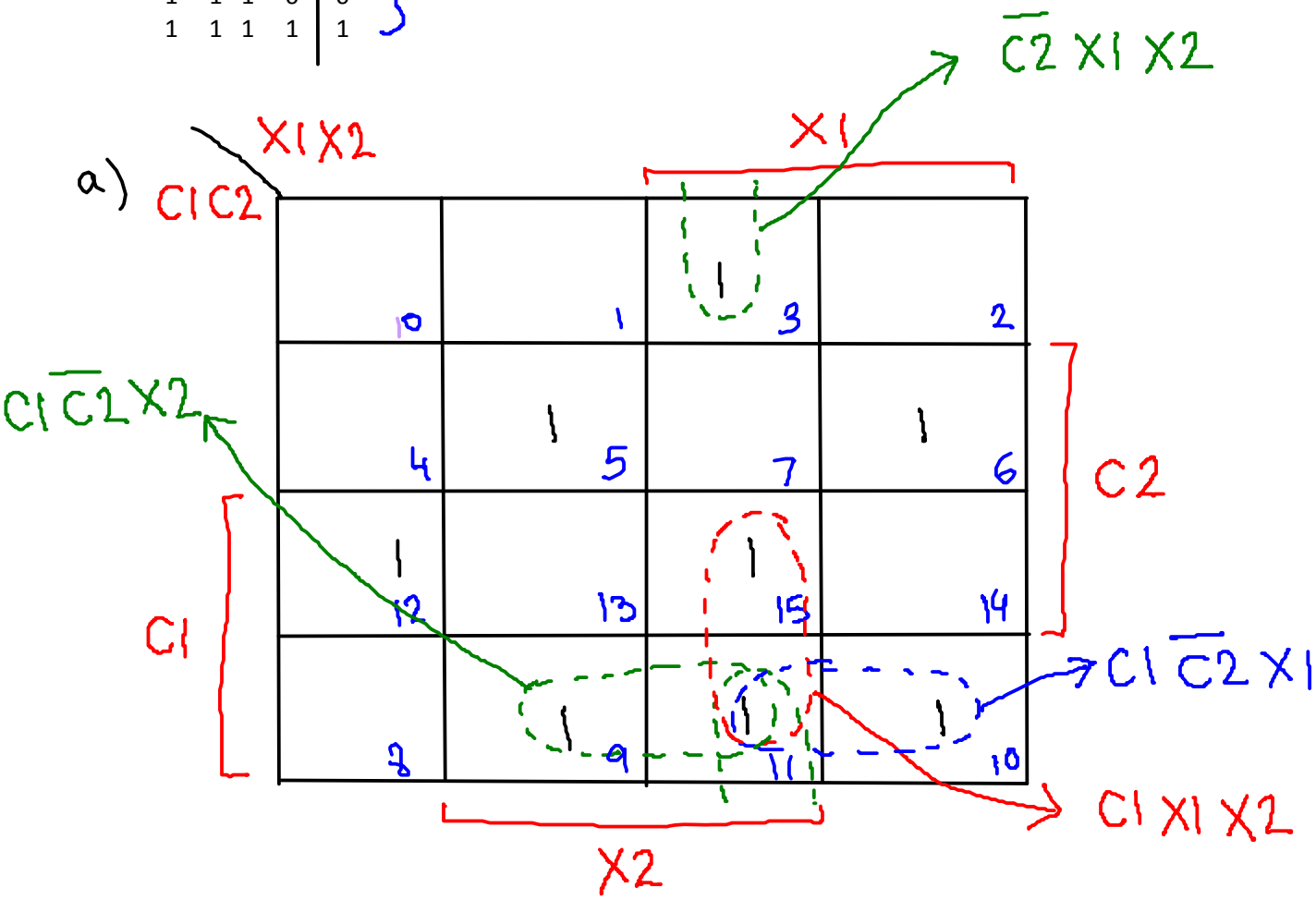
(a) Derive a truth table for Z
(b) Use a Karnaugh map to express Z as a simplified sum of product terms

Note: The Equivalence Operation is defined as:

X	Y	X $\equiv$ Y
0	0	1
0	1	0
1	0	0
1	1	1

C1	C2	X1	X2	Z	
0	0	0	0	0	$X1 \cdot X2$
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	1	
0	1	0	0	0	$X1 \oplus X2$
0	1	0	1	1	
0	1	1	0	1	
0	1	1	1	0	
1	0	0	0	0	$X1 + X2$
1	0	0	1	1	
1	0	1	0	1	
1	0	1	1	1	
1	1	0	0	1	$X1 \equiv X2$
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	1	

a)



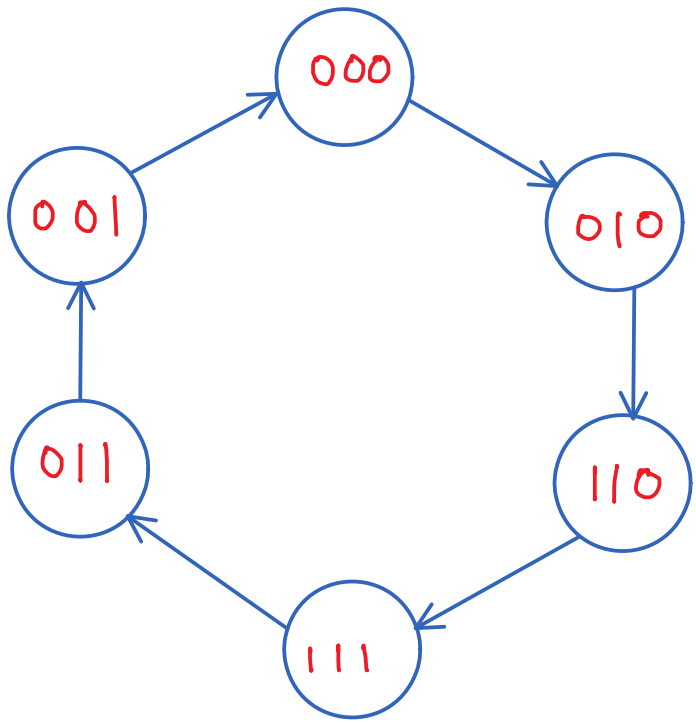
$$Z = C1\overline{C2}X2 + \overline{C2}X1X2 + C1X1X2 + C1\overline{C2}X1 +$$
$$C1C2\overline{X1}\,\overline{X2} + \overline{C1}C2\overline{X1}X2 + \overline{C1}C2X1\overline{X2}$$

Continued …

Problem 2 (50 points): Design a counter which counts in the sequence,

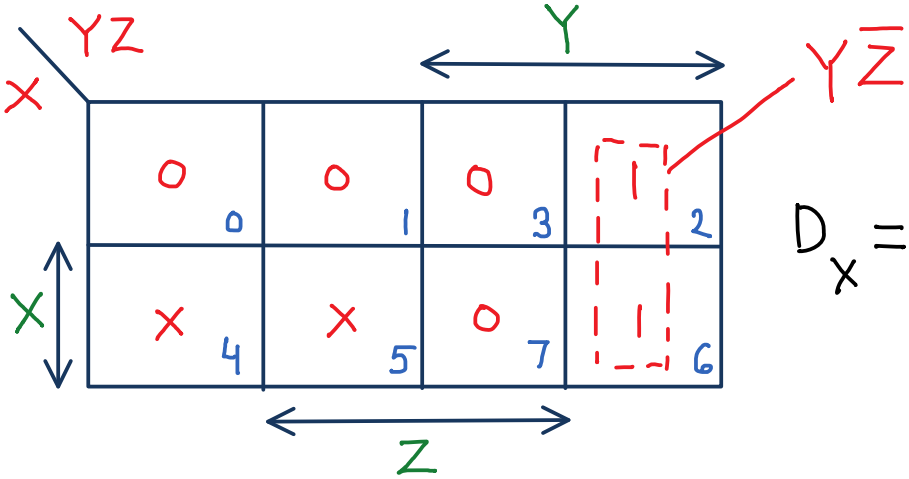
000 -> 010 -> 110 -> 111 -> 011 -> 001 -> repeat,

using D flip-flops. Simplify the circuit as much as possible using k-maps.



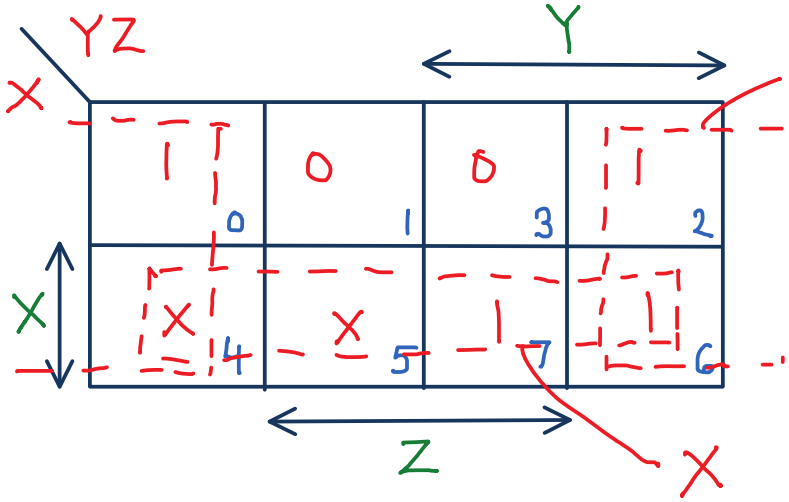
X	Y	Z	X^+	Y^+	Z^+	D_X	D_Y	D_Z
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	0
0	1	0	1	1	0	1	1	0
0	1	1	0	0	1	0	0	1
1	0	0	X	X	X	X	X	X
1	0	1	X	X	X	X	X	X
1	1	0	1	1	1	1	1	1
1	1	1	0	1	1	0	1	1

D_X



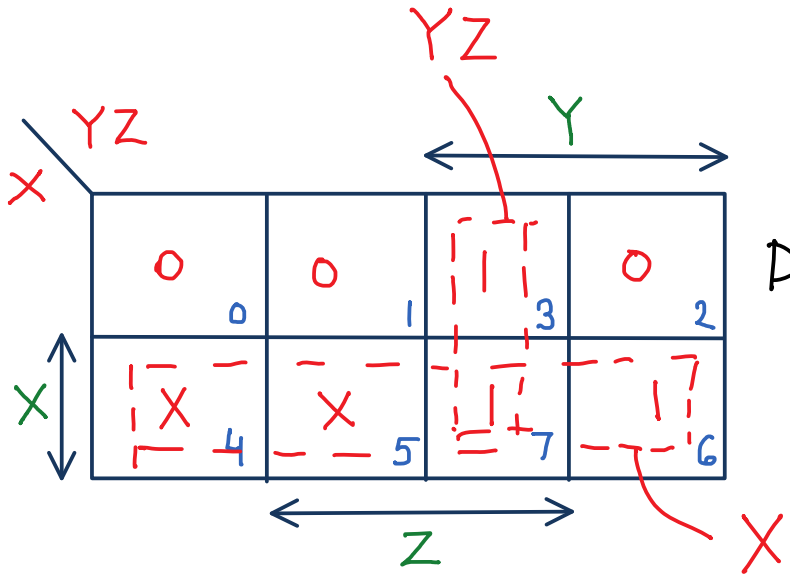
$D_X = Y\bar{Z}$

D_Y



$D_Y = \bar{Z} + X$

D_Z

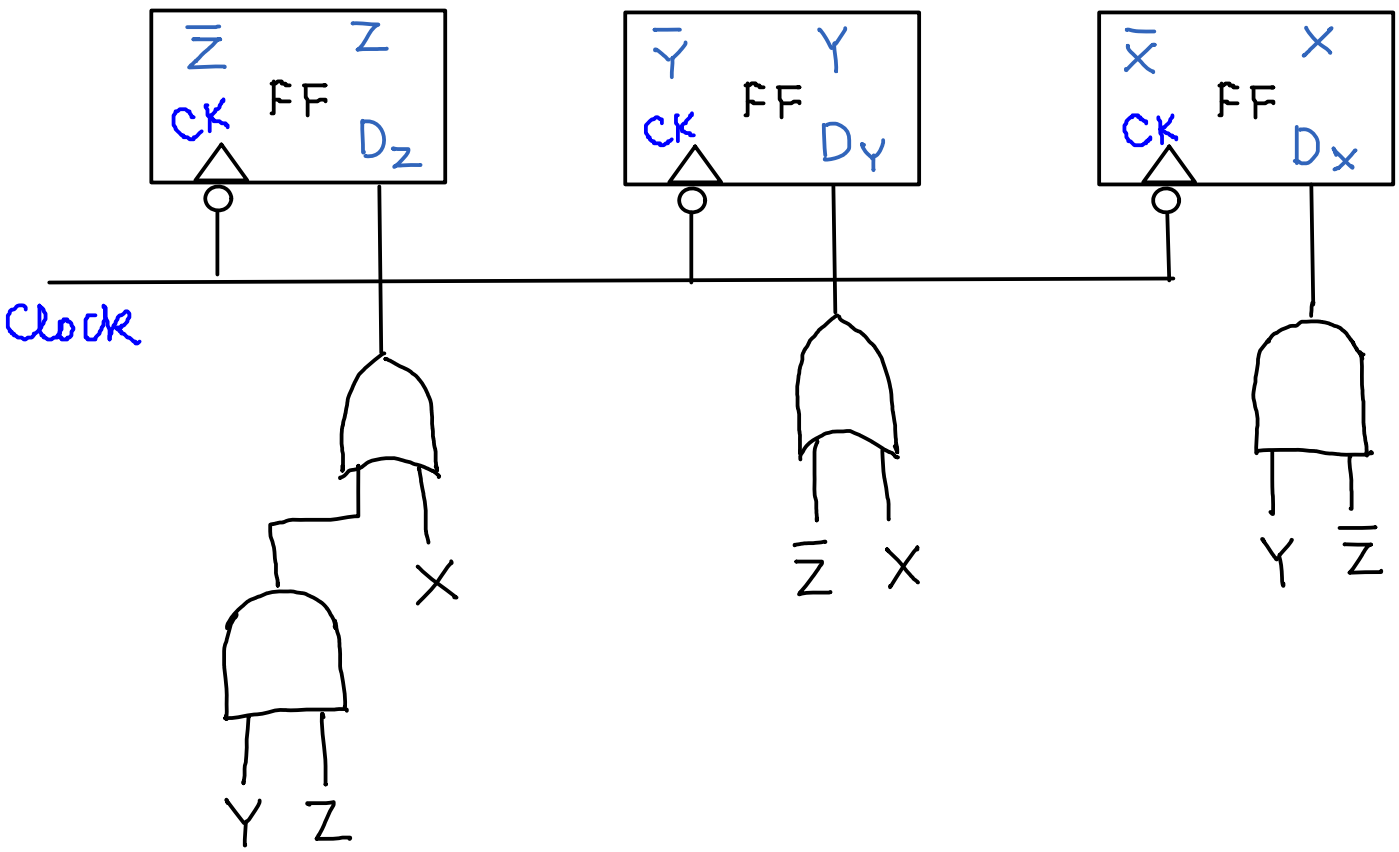


$D_Z = X + YZ$

$$D_X = Y\bar{Z}$$

$$D_Y = \bar{Z} + X$$

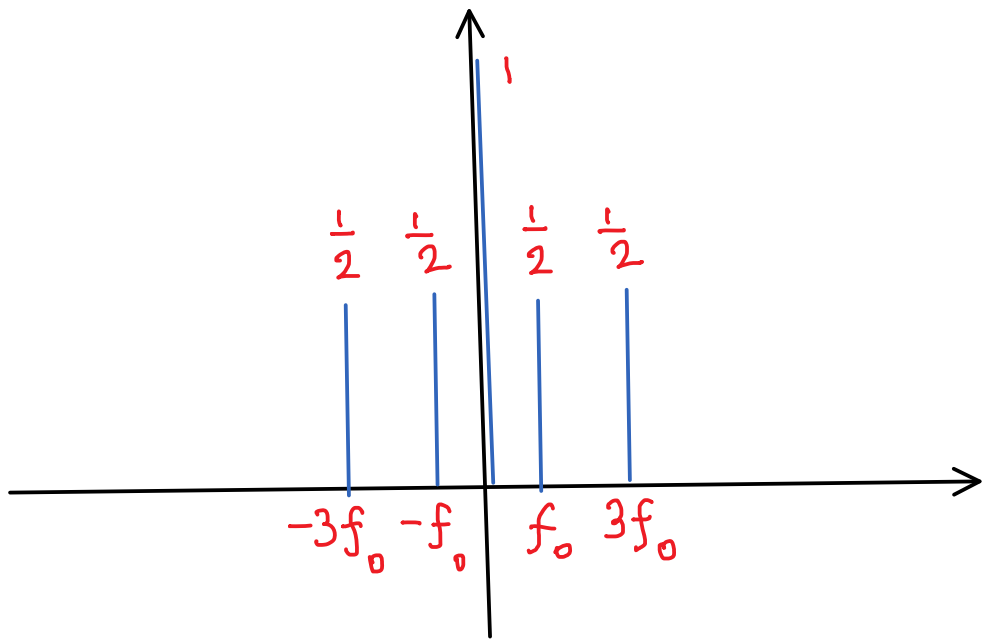
$$D_Z = X + YZ$$



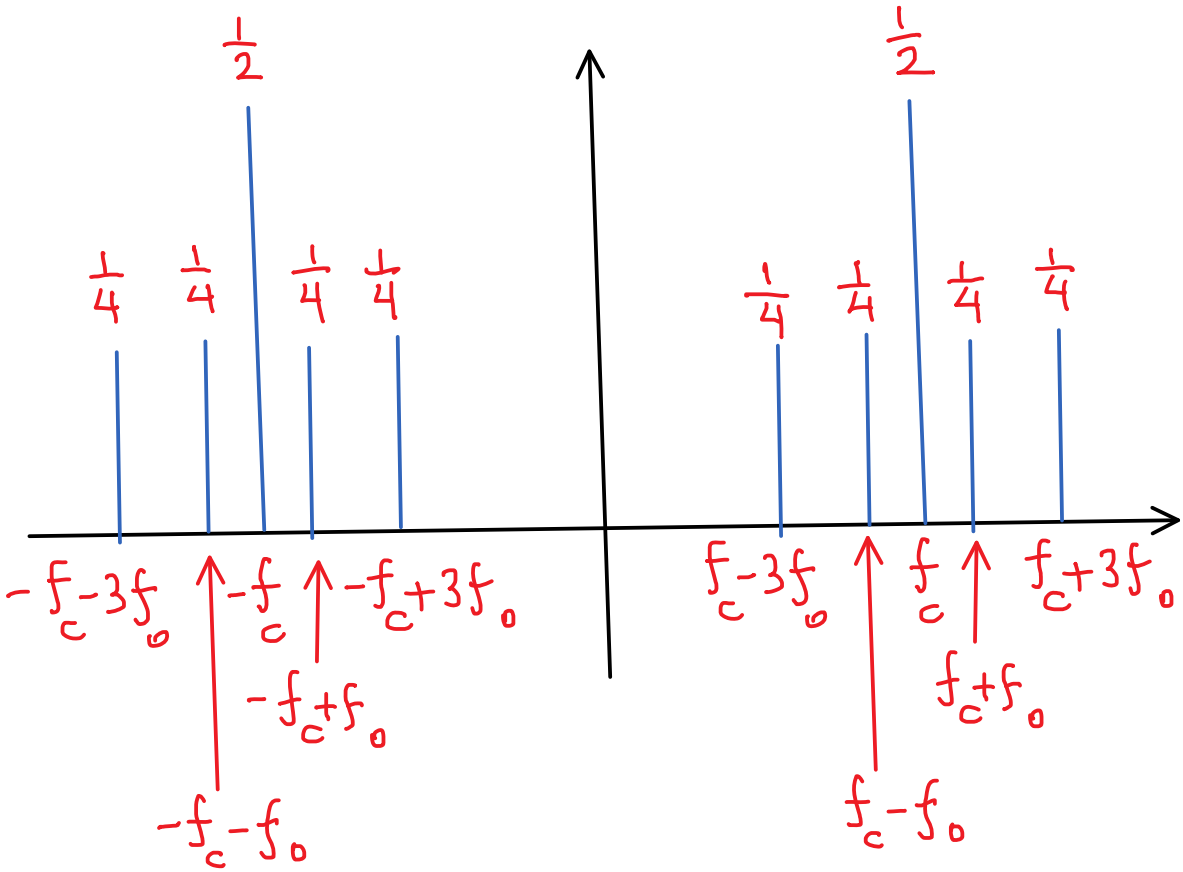
Problem 3 (50 points): Determine and plot the two sided magnitude spectrum (magnitude vs. frequency in Hz) of the signal given below (f0 = 20Hz, fc = 100Hz):

$$\underbrace{[\ 1 + \cos(2*pi*f0*t) + \cos(6*pi*f0*t)\ ]*\cos(2*pi*fc*t)}_{v(t)}$$

Spectrum of $v(t)$



Spectrum of $v(t)\cos(2\pi f_c t)$



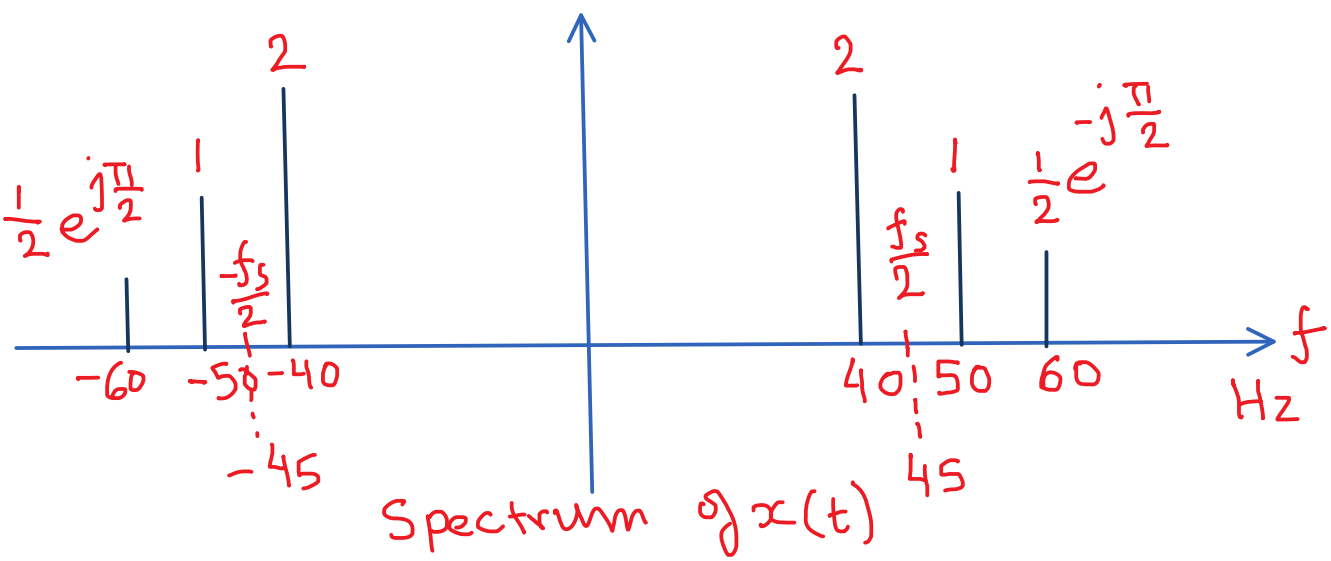
Problem 4 (50 points): A signal x(t) given below is sampled at a rate of 90 Hz by using a Continuous-to-Discrete converter. The sampled signal is then passed through a Discrete-to-Continuous converter to get a signal y(t).

- Determine the signal y(t).
- Determine the spectrum of x(t) and y(t).

Note that the spectrum for certain frequencies could be a complex number.

$$x(t) = 4\cos(80\pi t) + 2\cos(100\pi t) + \sin(120\pi t)$$

$$= 4\cos(2\pi \times 40t) + 2\cos(2\pi \times 50t) + \cos\left(2\pi \times 60t - \frac{\pi}{2}\right)$$



Black lines are the magnitude spectrum

The ±50 and ±60 Hz lines need to be folded into the fundamental frequency range

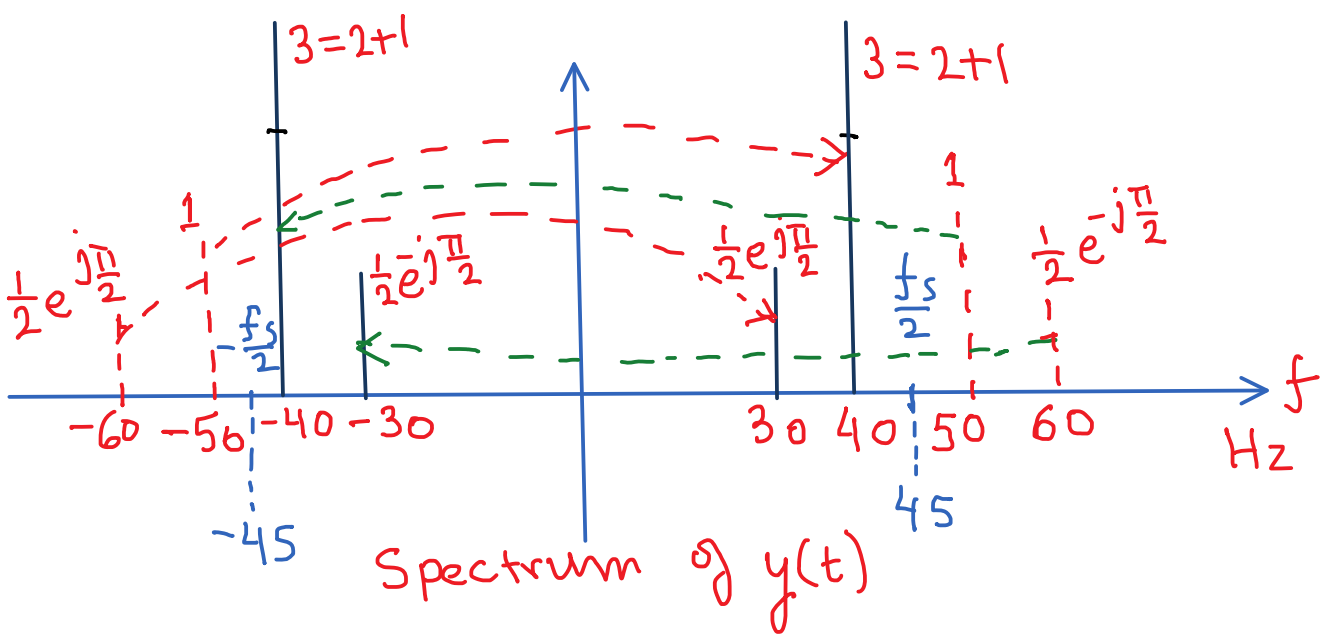
$50\,Hz \longrightarrow 50 - f_s = 50 - 90 = -40\,Hz$

$60\,Hz \longrightarrow 60 - f_s = 60 - 90 = -30\,Hz$

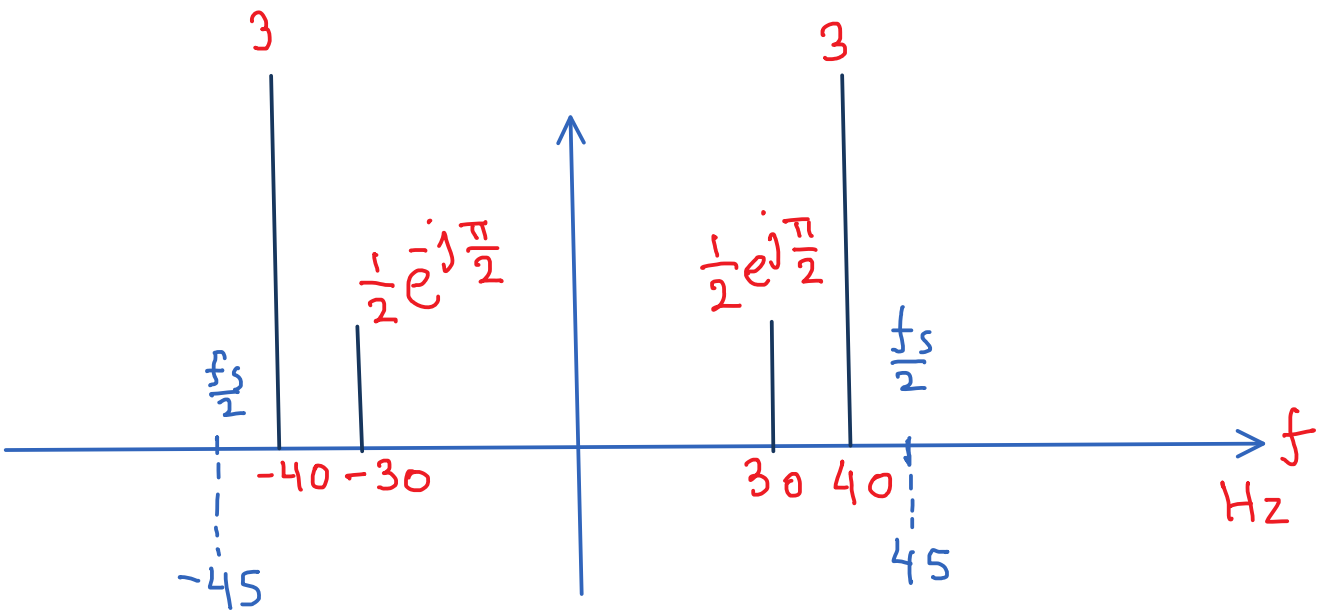
$-50\,Hz \longrightarrow -50 + f_s = -50 + 90 = 40\,Hz$

$-60\,Hz \longrightarrow -60 + f_s = -60 + 90 = 30\,Hz$

Note that -50Hz lands on top of 40Hz therefore the folded spectrum line needs to be added to the already existing spectrum line at 40 Hz (same with the 50Hz line added to the -40 Hz line)



Spectrum of y(t)



spectrum of $y(t)$

$$y(t) = \cos\left(2\pi \times 30t + \frac{\pi}{2}\right) + 6\cos(2\pi \times 40t)$$

$$= \cos(60\pi t + 90^\circ) + 6\cos(80\pi t)$$

magnitude spectrum of y(t)